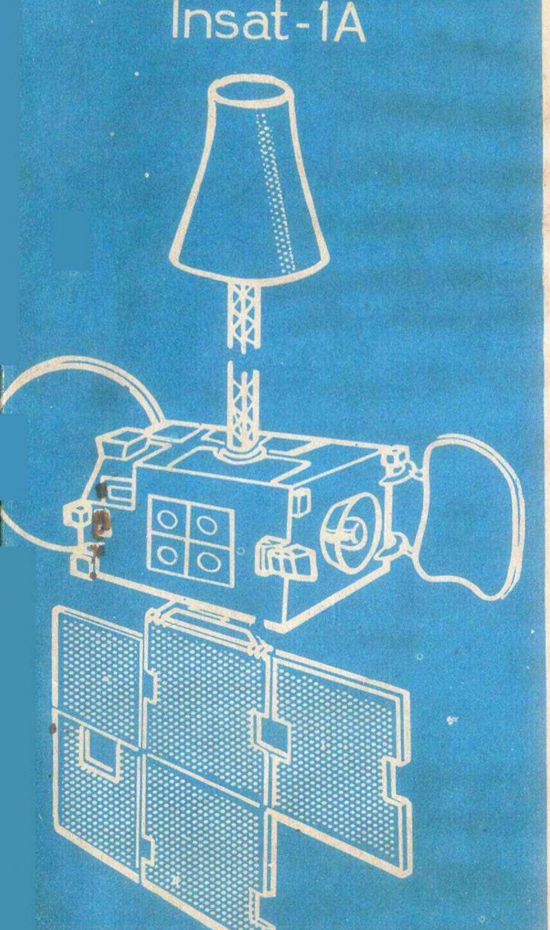
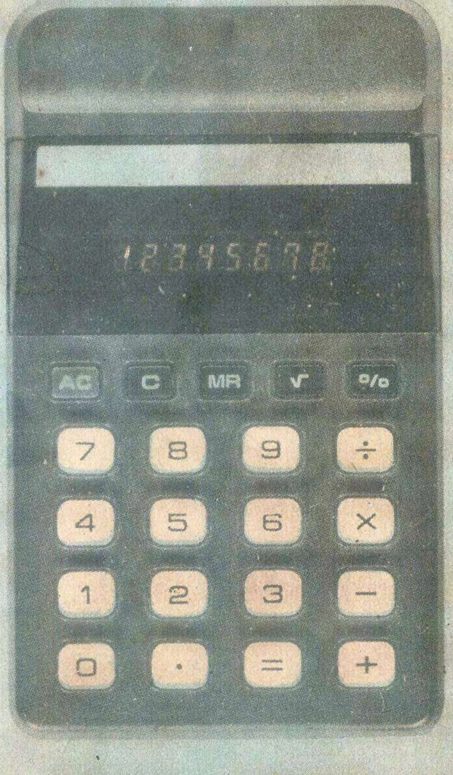
ALGEBRA

STANDARD X





ALGEBRA

STANDARD X

MAHARASHTRA STATE
BOARD OF SECONDARY AND HIGHER SECONDARY EDUCATION,
PUNE 411 010

First Edition: 1984
Reprint : 1992

© Maharashtra State Board of Secondary and Higher Secondary Education, Pune 411 010.

"This book is prepared by the Committee of Writers under the guidance of the Board of Studies and the same is edited."

Members of the Board of Studies : Dr. B.S. Fadnis (convener)
Prof. M.S. Huzurbazar (Convener) [From 27-7-84]
Shri S.A. Kathawate
Shri P.G. Relekar
Prof. B.B. Shinde
Dr. D.J. Tembhare
Prof. B.A. Chopade
Shri B.B. Patil
Shri L.E. Patil
Shri P.J. Nikam

Committee of Writers : Shri D.R. Jariwala
Shri D.S. Tambolkar
Mrs. Kishori Deshpande

Co-ordinator : Shri V.D. Godbole

Publisher : Shri. D.R. Gogate ,Secretary,
Maharashtra State Board of Secondary and
Higher Secondary Education, Pune 411 010.

Production Distribution : Maharashtra State Bureau of Textbook
Production & Curriculum Research, Pune
411 004.

Paper : 61 × 86 cm white printing paper

Printer : M/s. Sudha A. Printers - Binders, 140 A to Z
Industrial Estate, Lower Parel, Bombay 400 013.

Print order : N/Ptg/T.B. - 7273 (50,000)

PLEDGE

"India is my country. All Indians are my brothers and sisters.

I love my country and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness."

PREFACE

Taking into account the needs, the age groups and the understanding capacity of students belonging to all strata of society, the Maharashtra State Board of Secondary and Higher Secondary Education, having given a second look to the objectives of mathematics courses at all school levels, has reframed the syllabus of Mathematics from Std. VIII to X.

The text books of mathematics for Std. VIII and Std. IX, which were prepared as per the revised syllabus, were already published. In continuation of that series, the text book of mathematics for Std. X is prepared. It is hoped that the book finds approval from students as well as from teachers.

While writing this book main attempt has been to make students aware of the fact that mathematics is closely related with day-to-day life. Attempt is also made to achieve the following objectives.

* To awaken and encourage the inquisitive mind in students.
* To develop students' ability of logical thinking, so as to enable them to put ideas in precise language and to create liking towards it.
* To make them appreciate the interrelation of ideas which are responsible for growth of Mathematics.
* To make them aware that mathematics is essential for deeper understanding of all sciences.
* To make them realise that mathematics has its own language which they should be able to understand, to speak, and to write.
* To develop the capacity of students to solve examples and problems.

The matter in this text book is so presented as to establish a good conversation between the content matter of Std. IX and Std X and to facilitate the revision of the main concepts learned in Std. IX to help students for better understanding of new concepts to be introduced in this text book.

Prior to publication the script of this textbook was sent to renowned educationists, to experienced teachers and to wellknown authors of books in mathematics and their criticism, suggestions and opinions were solicited. This textbook is finalized after taking into consideration their suggestions and criticism. The Board is whole heartedly thankful to all these persons for their sincere suggestions and constructive criticism. The Board is grateful to the editor, co-ordinator and members of the Board of Studies appointed by the Board; who have extended their co-operation by putting their hard work with devotion in the preparation of this book.

The Board also thanks the Director of the Maharashtra State Bureau of Textbook production and curriculum Research, for his co-operation in bringing out this book. Thanks are also due to the Controller of the Bureau for getting it printed in a nice and attractive form.

Chairman,

Maharashtra State Board of
Secondary and Higher Secondary Education
and Additional Director of Education
Maharashtra State, Pune-411010.

Pune : 411 010.
Date : 15, August, 1983
24, Shravana, 1905

CONTENTS

Sr. No.	Chapter	Page No.
1.	Indices	1
2.	Ratio and proportion	12
3.	Parcentages	37
4.	Partnership	49
5.	Graph	56
6.	Equations	89
7.	Quadratic Equations	118
8.	Polynomials	131
9.	Rational Algebraic Expressions ..	166
10.	Variation	180
11.	Shares	195
12.	Statistics	201
	Answers	217

Chapter 1

Indices

In your eighth and ninth standard you have studied numbers having indices. You are also familiar with terms such as base, index or exponent and power. Let us therefore revise briefly those properties of numbers with indices which are already known to you.

§ 1.1 Revision.

1. If m is a positive integer then
   $a^m = a \times a \times a \times a \ldots\ldots\ldots m$ times
2. If $a \neq 0$, then $a^0 = 1$
3. If $a \neq 0$, then $a^{-r} = \dfrac{1}{a^r}$ and $a^r = \dfrac{1}{a^{-r}}$.
4. Following properties hold when the index is a positive integer zero or a negative integer.
   (i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \div a^n = a^{m-n}$, $(a \neq 0)$
   (iii) $(a^m)^n \longrightarrow = a^{mn}$ (iv) $(a \times b)^m = a^m \times b^m$

Exercise 1.1

1. Fill in the blanks.

(1) $a \times a \times a \times a \times a \times a = a^x \quad \therefore \quad x = \square$

(2) $3^0 \times 2^7 = \square$

(3) $3^p = \dfrac{1}{3^7} \quad \therefore \quad p = \square$

(4) $7^2 \times 7^5 = 7^x \quad \therefore \quad x = \square$

(5) $\dfrac{2^9 \times 2^2}{2^6} = 2^x \quad \therefore \quad x = \square$

(6) $\left(\dfrac{5^2 \times 5^3}{7^8}\right)^0 = x \quad \therefore \quad x = \square$

(7) $x^{-3} \times x^{-5} = x^p \quad \therefore \quad p = \square$

(8) $\left(x^{-3} \times x^{-5}\right)^{-2} = x^n \quad \therefore \quad n = \square$

(9) $\frac{2^8 \times 2^{-5}}{2^3} = 2^x$ $\therefore$ $x = \square$ and $2^x = \square$

(10) $x^3 \times x^{-8} \times x = x^a$ $\therefore$ $a = \square$

(11) $4^2 \times 4^x \times 4^{-3} = 4^9$ $\therefore$ $x = \square$

(12) $\frac{3^5 \times 3^6}{3^y} = 3^8$ $\therefore$ $y = \square$

(13) $\left(x^3 \right)^p = x^{-12}$ $\therefore$ $p = \square$

(14) $t^{-5} \times t^q = t^{-13}$ $\therefore$ $q = \square$

(15) $(b^{-3})^2 = b^{-3+x}$ $\therefore$ $x = \square$

(16) $4 \times 2^2 = 2^m$ $\therefore$ $m = \square$

(17) $\frac{3^4 \times 3^{-2}}{3^{-5}} = 3^{4+x}$ $\therefore$ $x = \square$

(18) $a^{-12} \div a^{-6} = a^k$ $\therefore$ $k = \square$

2. Determine which of the numbers given in examples below is the larger and which is smaller.

(i) $(81)^{\frac{3}{4}}$, $(27)^{\frac{4}{3}}$ (ii) 2^{-4}, 4^{-2} (iii) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$, $\left(\frac{8}{27}\right)^{\frac{4}{3}}$

3. Find the values of—

(i) $(125)^{\frac{2}{3}}$ (ii) $(16)^{\frac{3}{4}}$ (iii) $(243)^{\frac{3}{5}}$ (iv) $(32)^{\frac{3}{2}}$

§ 1.2 Fractional Indices.

An index of a number may be a positive or a negative fraction. We have met with numbers with positive fractional indices in standard IX. Let us revise the meaning attributed to positive fractional index.

n–Roots of a number.

1. 3 Squared is 9. Therefore square root of 9 is 3. Symbolically

$3^2 = 9$ $\therefore$ $(9)^{\frac{1}{2}} = 3$

2. 3 cubed is 27. Therefore cube root of 27 is 3. Symbolically

$3^3 = 27$ $\therefore$ $(27)^{\frac{1}{3}} = 3$

3. 4th power of 5 is 625. Therefore 4th root of 625 is 5. symbolically

$5^4 = 625 \quad \therefore \; (625)^{\frac{1}{4}} = 5$

4. 8th power of 2 is 256. Therefore 8th root of 256 is 2. Symbolically

$2^8 = 256 \quad \therefore \; (256)^{\frac{1}{8}} = 2$

In general, if a is a positive real number, m is a positive integer and b is a real number such that $b^m = a$
then b is said to be the mth root of a.

So that $\quad b = a^{\frac{1}{m}}$

Exercise 1.2

1. State which root and of what number do the following indicate :

   (i) $5^{\frac{1}{8}}$ (ii) $(5 \times 2)^{\frac{1}{14}}$ (iii) $\left(\frac{4}{3}\right)^{\frac{1}{6}}$

2. Write in symbolic form :

   (i) third root of 5. (ii) twelfth root of 48

3. Find the value of :

   (i) $\left(4^{\frac{1}{5}}\right)^5$ (ii) $\left(21^{\frac{1}{4}}\right)^4$ (iii) $\left[\left(\frac{4}{7}\right)^{11}\right]^{\frac{1}{11}}$

§ 1.3 Positive Rational Indices :

Let us write in symbols ' the 4th power of the 3 rd root of 125 '

3rd root of $125 = 125^{\frac{1}{3}}$

4th power of this will be $\left(125^{\frac{1}{3}}\right)^4$

But $\left(125^{\frac{1}{3}}\right)^4$ is written as $125^{\frac{4}{3}}$ so that

$(125)^{\frac{4}{3}}$ will mean the 4th power of the 3rd root of 125.

Similarly,

1. $64^{\frac{2}{5}}$ = 2nd power of the 5th root of 64
2. $36^{\frac{5}{7}}$ = 5th power of the 7th root of 36
3. $100^{\frac{3}{5}}$ = 3ra power of the 5th root of 100

In general,

if a is a positive real number and p and q are positive integers, then $a^{\frac{p}{q}}$ will mean the pth power of the qth root of a.

In symbols, $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}} \right)^p$

We have already seen that the laws which hold true when indices are positive integers continue to hold true when indices are positive fractions.

Let us now see what meaning can be ascribed when fractional indices assume negative values.

We know that a^{-3} is $\frac{1}{a^3}$ and a^3 is $\frac{1}{a^{-3}}$ We first came across this property in the VIII standard and have used it often since then. We assume that the same meaning continues to hold when indices are negative rational numbers.

Then $a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}}$ and $a^{\frac{3}{4}} = \frac{1}{a^{-\frac{3}{4}}}$

If a is a real number and if p and q are positive integers, then

$$a^{-\frac{p}{q}} = \frac{1}{a^{\frac{p}{q}}}$$

Note that $a^{-\frac{p}{q}}$ and $a^{\frac{p}{q}}$ **are reciprocals of each other.**

§ 1.4 Rational Indices :

We have learnt to interpret the meaning of indices when they are positive or negative fractions. We will now verify that the laws of indices which are true for integral indices also hold true for positive and negative fractional indices.

Since fractions can both be positive as well as negative we will have to consider the following cases in which indices m and n fall :

(1) indices m and n are both positive fractions,

(2) one of m and n is positive and the other is negative,

(3) indices m and n are both negative.

The first case was covered when you studied laws of indices in the ninth standard. We will, therefore, verify that the laws hold in cases (2) and (3) above.

(1) If a is a positive real number and if one of the indices m and n is positive and the other is negative or both the indices are negative, then we shall verify that

$$a^m \times a^n = a^{m+n}$$

(i) Let us show that $16^{-\frac{3}{4}} \times 16^{\frac{1}{4}} = 16^{-\frac{3}{4}+\frac{1}{4}}$

$$\begin{aligned}
\text{L. H. S.} &= 16^{-\frac{3}{4}} \times 16^{\frac{1}{4}} \\
&= \frac{1}{16^{\frac{3}{4}}} \times 16^{\frac{1}{4}} \\
&= \frac{1}{\left(16^{\frac{1}{4}}\right)^3} \times 16^{\frac{1}{4}} \\
&= \frac{1}{2^3} \times 2 \\
&= \frac{1}{2^2} \\
&= \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
\text{R. H. S.} &= 16^{-\frac{3}{4}+\frac{1}{4}} \\
&= 16^{-\frac{1}{2}} \\
&= \frac{1}{(16)^{\frac{1}{2}}} \\
&= \frac{1}{4}
\end{aligned}$$

$\therefore$ L. H. S. = R. H. S.

(ii) Let us now show that $16^{-\frac{3}{4}} \times 16^{-\frac{1}{4}} = 16^{\left(-\frac{3}{4}\right)+\left(-\frac{1}{4}\right)}$

$$\begin{aligned}
\text{L. H. S.} &= 16^{-\frac{3}{4}} \times 16^{-\frac{1}{4}} \\
&= \frac{1}{16^{\frac{3}{4}}} \times \frac{1}{16^{\frac{1}{4}}} \\
&= \frac{1}{16}\left(\because \frac{3}{4}+\frac{1}{4} = 1\right)
\end{aligned}$$

$$\begin{aligned}
\text{R. H. S.} &= 16^{\left(-\frac{3}{4}\right)+\left(-\frac{1}{4}\right)} \\
&= 16^{-1} \\
&= \frac{1}{16}
\end{aligned}$$

$\therefore$ L. H. S. = R. H. S.

Exercise 1.3

1. **Fill in the blanks :**

(1) $5^{\frac{1}{2}} \times 5^{\frac{3}{2}} = 5^{\square} = \square$ (2) $14^{\frac{2}{3}} \times 14^{\frac{5}{3}} = 14^{\square}$

(3) $100^{\frac{2}{7}} \times 100^{\frac{3}{7}} = 100^{\square}$ (4) $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = a^{\square}$

(5) $5^{-\frac{1}{2}} \times 5^{\frac{7}{2}} = 5^{\square}$ (6) $20^{\frac{7}{3}} \times 20^{-\frac{2}{3}} = 20^{\square}$

(7) $10^{-\frac{5}{8}} \times 10^{\frac{8}{8}} = 10^{\square}$ (8) $3^{-\frac{2}{7}} \times 3^{-\frac{3}{7}} = 3^{\square}$

(9) $25^{-\frac{4}{9}} \times 25^{-\frac{7}{9}} = 25^{\square}$ (10) $a^{-\frac{14}{15}} \times a^{-\frac{4}{15}} = a^{\square}$

(11) $15^{-\frac{2}{3}} \times 15^{\frac{1}{6}} = 15^{\square}$ (12) $25^{\frac{3}{4}} \times 25^{-\frac{3}{8}} = 25^{\square}$

(13) $32^{\frac{5}{6}} \times 32^{-\frac{5}{3}} = 32^{\square}$ (14) $243^{\frac{3}{7}} \times 243^{-\frac{15}{28}} = 243^{\square}$

(15) $48^{-\frac{3}{5}} \times 48^{-\frac{3}{20}} \times 48^{\frac{3}{4}} = 48^{\square} = \square$

(16) $(\sqrt{2})^{-\frac{17}{6}} \times (\sqrt{2})^{\frac{8}{3}} \times (\sqrt{2})^{-\frac{3}{2}} = (\sqrt{2})^{\square} = \square$

2. Simplify :

(1) $128^{-\frac{2}{7}} \times 128^{-\frac{4}{7}}$ (2) $243^{-\frac{3}{5}} \times 243^{\frac{1}{5}} \times 243^{-\frac{4}{5}}$

(3) $32^{\frac{6}{5}} \times 32^{-\frac{4}{5}}$ (4) $64^{-\frac{1}{3}} \times 64^{\frac{2}{3}} \times 64^{-\frac{5}{3}}$

3. Simplify :

(1) $\dfrac{2^{\frac{2}{3}} \times 2^{\frac{4}{3}} \times 8^{\frac{1}{3}}}{16^{\frac{3}{4}}}$ (2) $\dfrac{81^{-\frac{3}{4}} \times 27^{\frac{2}{3}}}{9^{-\frac{1}{2}}}$

(3) $\dfrac{125^{-\frac{2}{3}} \times 25^{\frac{3}{2}}}{625^{-\frac{3}{4}}}$ (4) $\dfrac{1024^{\frac{7}{10}} \times 256^{-\frac{3}{4}}}{64^{-\frac{4}{3}}}$

4. Find the value of x :

(1) $a^{-\frac{1}{3}} \times a^{\frac{2}{3}} = a^{x}$ (2) $a^{\frac{1}{2}} \times a^{-\frac{5}{2}} = a^{x}$

(3) $a^{-\frac{2}{5}} \times a^{-\frac{8}{5}} = a^{x}$ (4) $a^{-\frac{2}{9}} \times a^{x} = a^{\frac{5}{9}}$

(5) $a^{x} \times a^{-\frac{2}{7}} = a^{-\frac{3}{7}}$ (6) $a^{x} \times a^{\frac{3}{5}} = a^{\frac{1}{5}}$

(2) If a is a positive real number and m and n are positive or negative fractions, then $a^{m} \div a^{n} = a^{m-n}$

Let us verify this law.

(i) Let us show that $8^{-\frac{4}{3}} \div 8^{\frac{2}{3}} = 8^{\left(-\frac{4}{3}\right)-\left(\frac{2}{3}\right)}$

L. H. S. $= 8^{-\frac{4}{3}} \div 8^{\frac{2}{3}}$

$= 8^{-\frac{4}{3}} \times \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{8^{\frac{4}{3}}} \times \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{8^{2}} \left(\because \frac{4}{3} + \frac{2}{3} = 2\right)$

R. H. S. $= 8^{\left(-\frac{4}{3}\right)-\left(\frac{2}{3}\right)}$

$= 8^{-\frac{6}{3}}$

$= 8^{-\frac{2}{3}}$

$= 8^{\frac{1}{2}}$

$\therefore$ L. H. S. = R. H. S.

(ii) We now show that $81^{-\frac{3}{4}} \div 81^{-\frac{5}{4}} = 81^{-\frac{3}{4}-\left(-\frac{5}{4}\right)}$

L. H. S. $= 81^{-\frac{3}{4}} \div 81^{-\frac{5}{4}}$

$= \frac{1}{81^{\frac{3}{4}}} \times \frac{1}{81^{-\frac{5}{4}}}$

$= \frac{1}{81^{\frac{3}{4}}} \times 81^{\frac{5}{4}}$

$= 81^{\frac{5}{4}-\frac{3}{4}}$

$= 81^{\frac{1}{2}}$

$= 9$

R. H. S. $= 81^{-\frac{3}{4}-\left(-\frac{5}{4}\right)}$

$= 81^{-\frac{3}{4}+\frac{5}{4}}$

$= 81^{\frac{1}{2}}$

$= 9$

$\therefore$ L. H. S. = R. H. S.

Exercise 1.4

1. Fill in the blanks :

(1) $28^{\frac{6}{7}} \div 28^{\frac{3}{7}} = 28^{\square}$

(2) $2^{-\frac{15}{4}} \div 2^{-\frac{11}{4}} = 2^{\square}$

(3) $25^{-\frac{17}{2}} \div 25^{-\frac{15}{2}} = 25^{\square}$

(4) $20^{\frac{7}{9}} \div 20^{\frac{4}{9}} = 20^{\square}$

(5) $64^{-\frac{5}{12}} \div 64^{-\frac{7}{12}} = 64^{\square}$

(6) $a^{\frac{5}{8}} \div a^{-\frac{13}{8}} = a^{\square}$

(7) $b^{-\frac{17}{24}} \div b^{\frac{11}{24}} = b^{\square}$

(8) $c^{-\frac{1}{2}} \div c^{\frac{5}{2}} = c^{\square}$

(9) $8^{\frac{1}{3}} \div 8^{-\frac{2}{3}} = 8^{\square}$

(10) $15^{-\frac{2}{5}} \div 15^{\frac{3}{10}} = 15^{\square}$

(11) $49^{\frac{1}{2}} \div 49^{-\frac{3}{8}} = 49^{\square}$ (12) $a^{\frac{7}{24}} \div a^{\frac{5}{8}} = a^{\square}$

(13) $c^{-\frac{3}{5}} \div c^{-\frac{2}{25}} = c^{\square}$ (14) $m^{-\frac{6}{7}} \div m^{-\frac{4}{21}} = m^{\square}$

2. Simplify :

(1) $64^{-\frac{5}{6}} \div 64^{-\frac{1}{6}}$ (2) $243^{\frac{1}{5}} \div 243^{\frac{2}{5}}$

(3) $1000^{-\frac{1}{3}} \div 1000^{\frac{4}{3}}$ (4) $625^{\frac{7}{4}} \div 625^{-\frac{1}{2}}$

3. Simplify :

(1) $25^{\frac{1}{3}} \times 5^{\frac{1}{3}} \div 125^{-\frac{1}{3}}$ (2) $2^{\frac{4}{3}} \times 8^{-2} \div 4^{\frac{1}{6}}$

(3) $81^{-\frac{4}{5}} \div 27^{-\frac{1}{5}} \times 3^{-\frac{2}{5}}$ (4) $\left(\frac{1}{4}\right)^{-\frac{1}{2}} \times \left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{16}\right)^{\frac{5}{2}}$

4. Find the value of x :

(1) $a^{\frac{2}{3}} \div a^{x} = a^{\frac{1}{6}}$ (2) $a^{-\frac{8}{5}} \div a^{\frac{2}{5}} = a^{x}$

(3) $a^{-x} \div a^{\frac{1}{4}} = a^{\frac{1}{8}}$ (4) $a^{-\frac{5}{2}} \div a^{x+1} = a^{-\frac{7}{2}}$

(3) We will now verify that

$$(a^{m})^{n} = a^{mn}$$

where a is a real positive number and m and n are any fractions.

(i) We now show that $\left(8^{-\frac{2}{3}}\right)^{\frac{1}{2}} = 8^{-\frac{2}{3}\times\frac{1}{2}}$

$$\text{L. H. S.} = \left(8^{-\frac{2}{3}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{8^{\frac{2}{3}}}\right)^{\frac{1}{2}}$$

$$= \frac{1}{8^{\frac{1}{3}}} \left(\because \tfrac{2}{3}\times\tfrac{1}{2}=\tfrac{1}{3}\right)$$

$$= \frac{1}{2}$$

$$\text{R. H. S.} = 8^{-\frac{2}{3}\times\frac{1}{2}}$$

$$= 8^{-\frac{1}{3}}$$

$$= \frac{1}{8^{\frac{1}{3}}}$$

$$= \frac{1}{2}$$

$\therefore$ L. H. S. = R. H. S.

(ii) We now show that $\left[(625)^{-\frac{3}{4}} \right]^{-\frac{2}{3}} = (\ 625\ )^{(-\frac{3}{4}) \times (-\frac{2}{3})}$

$$\text{L. H. S.} = \left[(625)^{-\frac{3}{4}} \right]^{-\frac{2}{3}} \qquad \text{R. H. S.} = (\ 625\ )^{(-\frac{3}{4}) \times (-\frac{2}{3})}$$

$$= \left(\frac{1}{625^{\frac{3}{4}}} \right)^{-\frac{2}{3}} \qquad = (\ 625)^{\frac{1}{2}}$$

$$\qquad\qquad\qquad\qquad = 25$$

$$= \left(625^{\frac{3}{4}} \right)^{\frac{2}{3}}$$

$$= (625\ )^{\frac{1}{2}} \left(\because \tfrac{3}{4} \times \tfrac{2}{3} = \tfrac{1}{2} \right)$$

$$= 25$$

$\therefore$ L. H. S. = R. H. S.

Exercise 1.5

1. Fill in the blanks :

(1) $\left(3^{\frac{2}{3}} \right)^{\frac{6}{5}} = 3^{\square}$ (2) $\left(2^{-\frac{4}{9}} \right)^{\frac{3}{2}} = 2^{\square}$

(3) $\left(32^{-\frac{5}{7}} \right)^{-\frac{3}{10}} = 32^{\square}$ (4) $\left(9^{\frac{3}{10}} \right)^{-\frac{6}{11}} = 9^{\square}$

(5) $\left(10^{-\frac{8}{3}} \right)^{\frac{6}{7}} = 10^{\square}$ (6) $\left(5^{-\frac{21}{22}} \right)^{\frac{11}{7}} = 5^{\square}$

2. Simplify :

(1) $\left(64^{\frac{1}{2}} \right)^{-3}$ (2) $\left(125^{-6} \right)^{\frac{1}{3}}$

(3) $\left(512^{-\frac{2}{3}} \right)^{-\frac{5}{3}}$ (4) $\left(343^{-\frac{1}{2}} \right)^{-2}$

3.

(1) $\left(4^{\frac{2}{3}} \right) \left(8^{\frac{1}{9}} \right)^{5}$ (2) $\left(9^{-\frac{1}{2}} \times 81^{-\frac{3}{4}} \right)^{\frac{1}{4}}$

(3) $25^{-\frac{4}{3}} \times \left(5^{-\frac{1}{9}} \right)^{3} \div \left(625 \right)^{-\frac{3}{4}}$

4. Find the value of x :

(1) $\left(a^{-\frac{2}{3}} \right)^{-\frac{3}{4}} = a^{x}$ (2) $\left(a^{x} \right)^{-\frac{7}{3}} = a^{\frac{14}{3}}$

(3) $\left(a^{-\frac{4}{5}}\right)^{x} = a^{-\frac{2}{15}}$ (4) $\left(a^{x+1}\right)^{\frac{5}{2}} = a^{-\frac{3}{2}}$

(4) Now we verify that if a and b are positive real numbers and m is a negative fraction, then $(a \times b)^{m} = a^{m} \times b^{m}$

We show that $(8 \times 125)^{-\frac{5}{3}} = 8^{-\frac{5}{3}} \times 125^{-\frac{5}{3}}$

L. H. S. $= (8 \times 125)^{-\frac{5}{3}}$

$$= (1000)^{-\frac{5}{3}}$$

$$= \frac{1}{(1000)^{\frac{5}{3}}}$$

$$= \frac{1}{\left(1000^{\frac{1}{3}}\right)^{5}}$$

$$= \frac{1}{10^{5}}$$

$$= \frac{1}{100000}$$

H. H. S. $= 8^{-\frac{5}{3}} \times 125^{-\frac{5}{3}}$

$$= \frac{1}{8^{\frac{5}{3}}} \times \frac{1}{125^{\frac{5}{3}}}$$

$$= \frac{1}{\left(8^{\frac{1}{3}}\right)^{5}} \times \frac{1}{\left(125^{\frac{1}{3}}\right)^{5}}$$

$$= \frac{1}{2^{5}} \times \frac{1}{5^{5}}$$

$$= \frac{1}{(2 \times 5)^{5}}$$

$$= \frac{1}{100000}$$

$\therefore$ L. H. S. = R. H. S.

Exercise 1.6

1. Fill in the blanks :

(1) $(27 \times 8)^{\frac{3}{4}} = 27^{\square} \times 8^{\square}$

(2) $(5 \times 21)^{-\frac{8}{3}} = 5^{\square} \times 21^{\square}$

(3) $(9 \times 38)^{-\frac{7}{2}} = 9^{\square} \times 38^{\square}$

2. Simplify :

(i) $\left(125 \times \frac{1}{64}\right)^{-\frac{1}{3}}$ (ii) $\left(64 \times \frac{1}{49} \times 196\right)^{-\frac{1}{2}}$

§ 1.5

It can be easily verified that the laws of indices hold true when either one or both of the indices m and n are zero. From this we can now say that the following laws of indices hold true when the indices m and n are any rational numbers.

(i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$ (iv) $(a \times b)^m = a^m \times b^m$

Miscellaneous Exercise - 1

1. Fill in the blanks :

(1) $8^{\frac{16}{3}} \times 5^{\frac{16}{3}} \times 40^{-\frac{2}{3}} = 40^{\square}$

(2) $10^{-\frac{2}{5}} \times 2^{-\frac{4}{5}} \times 5^{\frac{3}{5}} = 5^{\square} \times 2^{\square}$

(3) $48^{-\frac{2}{7}} \times 16^{-\frac{5}{7}} \times 3^{-\frac{5}{7}} = 48^{\square}$

(4) $\left(a^{-\frac{3}{4}} \times b^{\frac{5}{3}}\right)^{\frac{12}{7}} = a^{\square} \times b^{\square}$

2. Simplify :

(1) $3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}$ (2) $6^{\frac{6}{5}} \times 3^{-\frac{1}{5}} \div 2^{\frac{1}{5}}$

(3) $216^{\frac{5}{3}} \div 9^{\frac{3}{2}} \times 32^{-\frac{3}{5}}$ (4) $50^{-\frac{1}{2}} \times 1000^{-\frac{4}{3}} \times 25^{\frac{3}{2}} \times 2^{-\frac{1}{2}}$

(5) $\dfrac{32^{\frac{2}{5}} \times 4^{-\frac{1}{2}} \times 8^{\frac{1}{3}}}{2^{-2} \div 64^{-\frac{1}{3}}}$

3. Find the values of x, y and z in the following :

(1) $\left(a^{\frac{2}{3}}b^{\frac{1}{4}} \times b^{-\frac{1}{3}}c^{-\frac{1}{2}} \times a^{\frac{3}{4}}c^{\frac{1}{3}}\right)^{-\frac{3}{4}} = a^x b^y c^z$

(2) $\left[\dfrac{a^{-\frac{2}{5}}b^2c^{\frac{3}{4}}}{a^{\frac{1}{2}}b^{-\frac{1}{3}}c^{-2}}\right]^{\frac{2}{3}} = a^x b^y c^z$

4. It is obvious that if a^m and a^n (where $a \neq 0$, $a \neq 1$) are equal, then we should have $m = n$. Using this property find the value of x in the following :

(1) $3^{2x+1} = 3^{x-1}$ (2) $5^{x+3} = 25^{3x-4}$

(3) $4^{3x-1} = 8^{x+3}$

● ● ● ●

Chapter 2

Ratio and proportion

§ 2.1 Comparison :

In every day life we are reqaired to compare one thing with another. For example, values of things, weights of objects, sizes of objects etc. We may be required to compare marks of studends or their heights and ages. We can make this comparison only with the help of numbers. In such comparisons, numbers we come across are naturally only the positive numbers.

If marks obtained by Satish and Ramesh in Arithmetic are respectively 40 and 60, we may compare these marks in the following two ways.

(1) We can say thal Ramesh has scored $(60-40)=20$ marks more than Satish. We have made this comparison by subtraction.

(2) We can also say that Ramesh has scored one and half times the marks scored by Satish. In this comparison we have performed division. $\left(60 \div 40 = \frac{60}{40} = \frac{3}{2}\right)$. This enables us to say how many times the marks of Ramesh are the marks obtained by Satish.

In this chapter we shall lay all the emphasis on the second type of comparison.

To say how many times a given number is in comparison to another number or to say what part a given number is in comparison to another is to state the ratio that holds between them. The numbers between which the ratio is stated are known as the terms of that ratio. Let us revise what you already know about ratio means.

To find a ratio which a number bears with another one is to divide the first number by another. The ratio of 15 and 18 is written and read as below.

	Written as	**Read as**
(1)	$\frac{15}{18}$	15 by 18
(2)	15 : 18	15 as to 18

§ 2.2 Standard units and ratios :

Comparison is made between thing, of the same type. Comparison between different kinds of things may look absurd. To compare marks between obtained by Hari in some examination with Megha's heigtht is simply ridiculous. So the ratio between a number showing marks and a number showing height would be meaningless. The term 'ratio' is meaningful only when things of the same kind are compared.

Another point that should be noted in this connection is that things compared, although of the same kind, must not be expressed in different units. For example, if John has 5 Rs. and Mahesh has 80 paise, it will be wrong to say that the ratio of the amount of money which John has, to the amount which Mahesh has, is 5 : 80. To be able to compare the amount each has by expressing the ratio between them, these amounts must be expressed in the same unit.

Now, 5 Rs. = 500 paise.

Therefore the ratio which the amount of John bears to the amount of Mahesh will be 500 : 80

Suppose that Sudha's height is 1·52 m and Smita's height is 138 cm. The ratio which Sudha's height bears to Smita's height will be 1·52 : 1·38

We may generalise the above ideas in relation to ratio between two real numbers.

If x and y are real numbers and y is different from zero then $\frac{x}{y}$ or $x : y$ is the ratio which x has to y. x and y are said to be the 1st and 2nd terms respectively of the ratio $x : y$.

We may now look at the term ratio from another angle. This point of view will help us to solve problems which arise in life.

Before we can measure anything a unit of measurement is first fixed. We then measure a thing in comparison to this standard unit. When we say that the mass of the thing is 5 kg, we are asserting that the mass of the thing is five times the mass contained in the unit of mass. This is also expressed by saying that the ratio which the unit of mass bears to the mass of a given thing is 1:5.

Now suppose that I have two metal bars. I wish to compare their lengths. To do this I start with fixing first the unit of length. If in comparison to this unit, if the length of one bar is 4 units and of another bar is 7 units, then the ratio which the length of the first bar bears with the length of the other bar is 4 : 7. I may also say that the first bar is $\frac{4}{7}$ part of the length of the other. Please note that the numbers 4 and 7 indicate how many times in length do the bars measure in comparison to the unit chosen.

When we say that 2 : 3 is the ratio between the measures of two things of the same kind, we also mean to say that the two things are respectively two and three times in measure in comparison to the standard unit. Sometimes the unit of measure in not specified. If this unit is denoted by x, we can say that the measures of things are $2x$ and $3x$ units respectively. Let us take a practical example to further clarify this point.

Ashok and Milind weigh 20kg and 30kg respectively. Let us fix 10 kg as the unit of measure. Then the weights of Ashok and Milind would respectively be twice and thrice the weight contained in the unit chosen (10 kg.). We may, therefore, say that the ratio between weights of Ashok and Milind is 2 : 3. But when we are provided with only the following information, say, the ratio between the weights is of Ashok and Milind is 2 : 3 and nothing is said about the unit of measurement, then in this case if x kg. is assumed to be the unit of weight, we may rightly say that Ashok's weight is $2x$ kg and Milind's weight is $3x$ kg.

We may generalise the above notion and compare more than two things of the same kind at a time.

When we say that measures of angles of a triangle are 2 : 3 : 4, we mean to say that they are respectively two, three and four times the standard unit. Assuming the unit to be x^0, the measures of those angles respectively will be $2x^0$, $3x^0$ and $4x^0$.

§ 2.3 Solved Examples :

1. Ages of Suhas and his father are as 3 : 8. If the sum of their ages is 55 years, find their ages.

Let the age of Suhas be $3x$ years. Since the ratio of Suhas's age to his fathar is $3:8$, his father's age would be $8x$.

$\therefore$ $3x + 8x = 55$

$\therefore$ $x = 11$

$\therefore$ Age of Suhas $= 3x = 3 \times 5 = 15$ years.

Father's age $= 8x = 8 \times 5 = 40$ years.

2. The measures of angles of a triangle are as $3 : 5 : 4$, find their measures.

Let the first angle have measure $= 3x^0$

$\therefore$ The second angle has measure $= 5x^0$

and third angle has measure $= 4x^0$

We know that the sum of measure of a triangle is 180^0.

$\therefore$ $3x + 5x + 4x = 180$

$\therefore$ $x = 15$

$\therefore$ Measure of the first angle $= 3x = 3 \times 15 = 45^0$

Measure of the second angle $= 5x = 5 \times 15 = 75^0$

and Measure of the third angle $= 4x = 4 \times 15 = 60^0$

Exercise 2.1

1. Pairs of numbers are given below. Find the ratio which the first number of the pair bears with its second number, in its lowest form

(i) 9, 14 (ii) 21, 15 (iii) 28, 14

(iv) 32, 24 (v) 51, 34 (vi) $\frac{3}{5}$, $\frac{8}{10}$

(vii) $\sqrt{3}$, $\sqrt{5}$ (viii) 1·5, 4·5 (ix) $\sqrt{75}$, $\sqrt{48}$

2. In examples given below find the ratio which the first quantity bears with the second.

(i) 9 cm. , 15 cm. (ii) 55 cm. , 2 m.

(iii) 77 Ps. , 11 Rs. (iv) 6 yrs. 2 m. , 64 yrs., 9 m.

(v) 45 seconds, 1 minute (vi) 0·5 Litre, 400 ml.

3. The ratio between the ages of mother and son is $5 : 2$. If the sum of their ages is 42 years, find their ages.

4. Ratio between two numbers is $4 : 7$. Their sum is 77. Find the numbers.

5. Two numbers have ratio $12 : 5$. Their difference is 35. Find the numbers.

6. Area of a rectangle is 29·4 sq. cm. The ratio of its sides is $5 : 3$. Find the length and breadth of the rectangle.

7. Measures of angles of a triangle are to each other as $3 : 4 : 2$. Find the measure of each angle.

8. If measures of angles of a quadrilateral are to each other as $3 : 4 : 5 : 6$. Find the measure of each angle.

9. If the ratio between the radii of two circles is $2 : 3$, find the ratio between the circumference of each circle.

10. If areas of two circles are to each other as $4 : 25$, find the ratio of their radii and their circumferences.

§ 2.4 Ordering the ratios :

For the purpose of this section ratios considered are ratios of positive real numbers only. For this reason this fact is not stated everytime we study such ratios.

To find which of the ratios $\frac{a}{b}$, $\frac{c}{d}$ is smaller, we subtract the second from the first. If the result is zero, two ratios would be equal ; if positive, the first ratio would be greater than the second; if negative, the first ratio would be smaller than the second.

Now, $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

So that when $\frac{ad-bc}{bd} = o$, then $ad - bc = o$, i. e. $ad = bc$.

So that (i) if $ad = bc$, we have $\frac{a}{b} = \frac{c}{d}$.

This property states that in order to show that $\frac{a}{b} = \frac{c}{d}$, it is enough to show that $ad = bc$.

(ii) when $\frac{ad - bc}{bd} > o$, i. e. when $ad - bc > o$ or when $ad > bc$, we have $\frac{a}{b} > \frac{c}{d}$

(iii) when $\frac{ad-bc}{bd} < o$, i. e. when $ad - bc < o$, or when $ad < bc$

we have $\frac{a}{b} < \frac{c}{d}$,

Of two given ratios if

Numerator of first X Denominator of second – Deno minator of first X Numerator of second

(i) is zero, then the ratios are equal.

(ii) is positive, then the first is greater than the second

(iii) is negative, then the first is smaller than the second.

§ 2.5 Solved examples :

1. Find which of ratios $\frac{9}{16}$ and $\frac{5}{12}$ is the larger.

In this example $a = 9$, $b = 16$, $c = 5$, $d = 12$

$\therefore\ a \times d = 9 \times 12 = 108$, $b \times c = 16 \times 5 = 80$.

So that, $a \times d > b \times c$,

$\therefore\ \frac{a}{b} > \frac{c}{d}$ $\qquad \therefore\ \frac{9}{16} > \frac{5}{12}$

2. Determine which of the two ratios $\frac{21}{25}$ and $\frac{7}{8}$ is the larger ?

In this example,

$21 \times 8 = 168$ and $25 \times 7 = 175$.

But– $168 < 175$. $\therefore\ \frac{21}{25} < \frac{7}{8}$

3. If 10 is added to each term of the ratio $\frac{18}{23}$ and a new ratio is thus obtained, state which of the old and the new ratios is the larger ?

We have to find which of the two ratios $\frac{18}{23}$ and $\frac{18+10}{23+10} = \frac{28}{33}$ is the larger.

Here, $18 \times 33 = 594$ and $23 \times 28 = 644$.

But $594 < 644$ $\therefore \frac{18}{23} < \frac{28}{33}$

4. If 5 is added to both the terms of the rate $\frac{25}{19}$, compare the old ratio with the new one.

We have to determine which of the ratios $\frac{25}{19}$ and $\frac{25+5}{19+5} = \frac{30}{24}$ is the larger.

Here, $25 \times 24 = 600$, $19 \times 30 = 570$.

But, $600 > 570 \quad \therefore \frac{25}{19} > \frac{30}{24}$

Study examples (3) and (4) carefully. In the ratio of example (3) denominator is greater than the numerator. In example (4) numerator is greater than the denominator. From this,

(i) If the same positive number is added to both the terms of a ratio in which denominator is larger than the numerator, the new ratio is greater than the first.

(ii) If the same positive number is added to both the terms of a ratio in which the numerator is larger than the denominator, the new ratio is smaller than the first.

5. If a new ratio is obtained by multiplying both the terms of a ratio $\frac{a}{b}$ by k, find which ratio is greater.

The ratio obtained by multiplying both terms of the ratio $\frac{a}{b}$ by k is $\frac{ak}{bk}$

$$\text{Now } \frac{a}{b} - \frac{ak}{bk} = \frac{abk - bak}{b(bk)} = 0$$

$$\therefore \frac{a}{b} = \frac{ak}{bk}$$

From this it is clear that a ratio equals the ratio obtained by multiplying both terms by k.

Similar statement can be made if both terms of a ratio are divided by the same number.

6. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{ak}{bk} = \frac{cl}{dl}$

Using example (5) we have,

$$\frac{a}{b} = \frac{ak}{bk}$$

Similarly $\frac{c}{d} = \frac{cl}{dl}$.

But it is given that $\frac{a}{b} = \frac{c}{d}$

$\therefore \frac{ak}{bk} = \frac{cl}{dl}$

7. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{ak}{bl} = \frac{ck}{dl}$

Given $\frac{a}{b} = \frac{c}{d}$. To show that $\frac{ak}{bl} = \frac{ck}{dl}$ we will have to show $(ak)(dl) = (ck)(bl)$

Now, $\frac{a}{b} = \frac{c}{d}$

$\therefore ad = bc$

$\therefore adkl = bckl$ (multiplying both sides by kl)

From this we have,

$(ak)(dl) = (bl)(ck)$

So that $\frac{ak}{bl} = \frac{ck}{dl}$

That the above property holds can be shown also as follows

From $\frac{a}{b} = \frac{c}{d}$ we have $\frac{a}{b} - \frac{c}{d} = 0$

If we can show that $\frac{ak}{bl} - \frac{ck}{dl} = 0$, we can conclude that

$\frac{ak}{bl} = \frac{ck}{dl}$

$$\text{Now } \frac{ak}{bl} - \frac{ck}{dl} = \frac{a}{b} \times \frac{k}{l} - \frac{c}{d} \times \frac{k}{l}$$

$$= \left(\frac{a}{b} - \frac{c}{d}\right) \times \frac{k}{l}$$

$$= 0 \times \frac{k}{l} = 0$$

From this, if $\frac{a}{b} = \frac{c}{d}$ then $\frac{ak}{bl} = \frac{ck}{dl}$.

Exercise 2.2

Fill in the blanks choosing proper sign from $>, =, <$.

(1) $\frac{16}{21} \square \frac{5}{7}$ (2) $\frac{1}{2} \square \frac{23}{47}$ (3) $\frac{5}{9} \square \frac{85}{153}$

(4) $\frac{31}{13} \square \frac{38}{50}$ (5) $\frac{22}{35} \square \frac{24}{37}$ (6) $\frac{45}{24} \square \frac{15}{8}$

(7) $\frac{9}{25} \square \frac{36}{100}$ (8) $\frac{5}{18} \square \frac{5+42}{18+42}$ (9) $\frac{11}{19} \square \frac{11+35}{19+35}$

(10) $\frac{5}{11} \square \frac{5-2}{11-2}$ (11) $\frac{41}{37} \square \frac{41-9}{37-9}$ (12) $\frac{m}{n} \square \frac{m \times l}{n \times l}$

(13) If $p < q$ then $\frac{p}{q} \square \frac{p+10}{q+10}$

(14) If $x \times y < z \times t$, then $\frac{x}{z} \square \frac{t}{y}$

(15) If $\frac{s}{t} > \frac{s+r}{t+r}$ and $s, r, t > 0$, then $s \square$

§ 2.6 Properties of equal ratios :

1. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

Since $\frac{a}{b} - \frac{c}{d}$ we have $b \times c = a \times d$

Dividing both sides by $a \times c$ we get

$$\frac{b \times c}{a \times c} = \frac{a \times d}{a \times c}$$

$$\therefore \frac{b}{a} = \frac{d}{c}$$

This property is known as Invertendo

2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$.

Since $\frac{a}{b} = \frac{c}{d}$ we have $ad = bc$

Dividing both sides by cd we have,

$$\frac{ad}{cd} = \frac{bc}{cd}$$

So that $\frac{a}{c} = \frac{b}{d}$.

This property is called as Alternendo

3. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$

We have, $\frac{a}{b} = \frac{c}{d}$

$$\therefore \quad \frac{a}{b} + 1 = \frac{c}{d} + 1$$

$$\therefore \quad \frac{a+b}{b} = \frac{c+d}{d}$$

This property is known as Componendo

We establish this property by another method.

To show $\frac{a+b}{b} = \frac{c+d}{d}$

we show $(a+b)d = (c+d)b$

Now, $(a+b)d = ad + bd$

$$= bc + bd \left(\because \frac{a}{b} = \frac{c}{d} \therefore ad = bc \right)$$

$$= (c+d)b$$

So that finally $\frac{a+b}{b} = \frac{c+d}{d}$.

Repeatedly using this property m times,

we have $\frac{a+mb}{b} = \frac{c+md}{d}$.

4. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

we have $\frac{a}{b} = \frac{c}{d}$

$$\therefore \quad \frac{a}{b} - 1 = \frac{c}{d} - 1$$

$$\therefore \quad \frac{a-b}{b} = \frac{c-d}{d}$$

This property is known as Dividendo

Repeatedly using this property m times,

we have $\frac{a-mb}{b} = \frac{c-md}{d}$

5. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

From $\frac{a}{b} = \frac{c}{d}$

by using componendo we have $\frac{a+b}{b} = \frac{c+d}{d}$......... (I)

Similarly using dividendo we havc $\frac{a-b}{b} = \frac{c-d}{d}$......... (II)

Dividig (i) by (ii) we get $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

This property is known as componendo and dividendo

We can use the above properties to simplify many tedious calculations. Study the following examples carefully.

1. If $\frac{x}{y} = \frac{5}{3}$, find the value of $\frac{3x+2y}{3x-2y}$

We have, $\frac{x}{y} = \frac{5}{3}$

$\therefore \quad \frac{3x}{2y} = \frac{15}{6}$

$\therefore \quad \frac{3x+2y}{3x-2y} = \frac{15+6}{15-6}$......by componendo and dividendo

$\therefore \quad \frac{3x+2y}{3x-2y} = \frac{21}{9} = \frac{7}{3}$

2. If $\frac{x^3+y^3}{x^3-y^3} = \frac{76}{49}$, find the value of $\frac{x}{y}$

$\frac{x^3+y^3}{x^3-y^3} = \frac{76}{49}$

$\therefore \quad \frac{(x^3+y^3)+(x^3-y^3)}{(x^3+y^3)-(x^3-y^3)} = \frac{76+49}{76-49}$...... (by componendo and dividendo)

$\therefore \quad \frac{2x^3}{2y^3} = \frac{125}{27}$

$\therefore \quad \frac{x^3}{y^3} = \frac{125}{27} \qquad \therefore \quad \frac{x}{y} = \frac{5}{3}$

3. Solve the equation $\dfrac{(3x+2)^2+(3x-2)^2}{(3x+2)^2-(3x-2)^2} = \dfrac{5}{4}$

$$\frac{(3x+2)^2+(3x-2)^2}{(3x+2)^2-(3x-2)^2} = \frac{5}{4}$$

$\therefore$ *By* componendo and dividendo

$$\frac{\left[(3x+2)^2+(3x-2)^2\right]+\left[(3x+2)^2-(3x-2)^2\right]}{\left[(3x+2)^2+(3x-2)^2\right]-\left[(3x+2)^2-(3x-2)^2\right]} = \frac{5+4}{5-4}$$

$$\therefore \quad \frac{2(3x+2)^2}{2(3x-2)^2} = \frac{9}{1} \quad \therefore \quad \frac{(3x+2)^2}{(3x-2)^2} = \frac{9}{1}$$

$$\therefore \quad \frac{3x+2}{3x-2} = \frac{3}{1} \quad \text{or} \quad \frac{3x+2}{3x-2} = \frac{-3}{1}$$

$\therefore$ Again by componendo and dividendo,

$$\frac{(3x+2)+(3x-2)}{(3x+2)-(3x-2)} = \frac{3+1}{3-1} \quad \text{or} \quad \frac{(3x+2)+(3x-2)}{(3x+2)-(3x-2)} = \frac{-3+1}{-3-1}$$

$$\therefore \quad \frac{6x}{4} = \frac{4}{2} \quad \text{or} \quad \frac{6x}{4} = \frac{-2}{-4}$$

$$\therefore \quad x = \frac{4}{2} \times \frac{4}{6} \quad \text{or} \quad x = \frac{-2}{-4} \times \frac{4}{6}$$

$$\therefore \quad x = \frac{4}{3} \quad \text{or} \quad x = \frac{1}{3}$$

4. Solve $\dfrac{x^2 - 3x + 2}{3x - 2} = \dfrac{x^2 - 5x + 4}{5x - 4}$

$$\frac{x^2 - 3x + 2}{3x - 2} = \frac{x^2 - 5x + 4}{5x - 4}$$

$\therefore$ by componendo

$$\frac{(x^2 - 3x + 2) + (3x - 2)}{3x - 2} = \frac{(x^2 - 5x + 4) + (5x - 4)}{5x - 4}$$

$$\therefore \quad \frac{x^2}{3x - 2} = \frac{x^2}{5x - 4}$$

Now, it can be easily verified that $x = 0$ is a solution of the equation.

If $x \neq 0$ then $x^2 \neq 0$

$\therefore$ Dividing both sides by x^2

$$\frac{1}{3x-2} = \frac{1}{5x-4}$$

$\therefore \quad 3x - 2 = 5x - 4$

$\therefore \quad 2 = 2x$

$\therefore \quad 1 = x$

$\therefore \quad x = 0$ or $x = 1$ is the solution of the given equation.

Exercise 2.3

1. If $\frac{x}{y} = \frac{7}{4}$ then find the value of $\frac{3x + 5y}{3x - 5y}$
2. If $\frac{p}{q} = \frac{2}{3}$, find the value of $\frac{5p + q}{5p - q}$
3. If $\frac{3a + 5b}{3a - 5b} = 5$, find the value of $\frac{a}{b}$
4. If $\frac{m}{n} = \frac{5}{4}$, find the value of $\frac{m^2 + n^2}{m^2 - n^2}$
5. If $a : b = 2 : 3$, evaluate $\frac{3a - 2b}{4a + 3b}$
6. Solve the following equations (x is a nonnegative number.)

(1) $\frac{(x+3)^2 + (x-3)^2}{(x+3)^2 - (x-3)^2} = \frac{37}{12}$

(2) $\frac{(2x+3)^3 + (2x-3)^3}{(2x+3)^3 - (2x-3)^3} = \frac{76}{49}$

(3) $\frac{x^2 - 2x + 3}{2x - 3} = 2$

(4) $\frac{x^2 - 3x + 1}{3x - 1} = \frac{x^2 - 2x + 5}{2x - 5}$

(5) $\frac{x^2 + 3x - 5}{3x - 5} = \frac{x^2 + 2x - 7}{2x - 7}$

(6) $\frac{\sqrt{x+5} + \sqrt{x-3}}{\sqrt{x+5} - \sqrt{x-3}} = \frac{3}{2}$

§ 2.8 An important property of equal ratios :

The ratios $\frac{6}{10}$ and $\frac{3}{5}$ are equal. If their denominators and numerators are added, we get $\frac{6+3}{10+5} = \frac{9}{15}$ as the new ratio. But

$\frac{9}{15}$ is also equal to $\frac{6}{10}$ or to $\frac{3}{5}$. From this we can state a new property of equal ratios

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{a+c}{b+d} = \frac{c}{d}$

That this property holds can be shown as follows.

We are given that $\frac{a}{b} = \frac{c}{d}$. To show $\frac{a}{b} = \frac{a+c}{b+d}$ it is enough to show $\frac{a}{b} - \frac{a+c}{b+d} = 0$

$$
\begin{aligned}
\text{Now, } \frac{a}{b} - \frac{a+c}{b+d} &= \frac{a(b+d) - b(a+c)}{b(b+d)} \\
&= \frac{ab + ad - ab - bc}{b(b+d)} \\
&= \frac{ad - bc}{b(b+d)} \\
&= \frac{0}{b(b+d)} \ldots\ldots \left(\because \frac{a}{b} = \frac{c}{d} \therefore ad = bc \right) \\
&= 0
\end{aligned}
$$

$$\therefore \frac{a}{b} = \frac{a+c}{b+d} = \frac{c}{d}$$

We can extend this property as follows.

Suppose, $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \ldots\ldots\ldots$

We start with $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

So that, $\frac{a+c}{b+d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$,

from this $\frac{a+c+e}{b+d+f} = \frac{g}{h} = \frac{a+c+e+g}{b+d+f+h}$

Continuing this mode of reasoning we have,

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \ldots\ldots\ldots\ldots$

then each ratio $= \dfrac{a+c+e+g\ \ldots\ldots}{b+d+f+h\ \ldots\ldots}$

You may verify the above by taking equal ratios as $\dfrac{10}{8}, \dfrac{25}{20}, \dfrac{5}{4}, \dfrac{70}{56}, \ldots\ldots$

We can extend the above property further.

We have seen that $\dfrac{a}{b} = \dfrac{a \times k}{b \times k}$

Therefore, if $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}\ \ldots\ldots$

Then each $= \dfrac{la}{lb} = \dfrac{mc}{md} = \dfrac{ne}{nf} = \ldots\ldots$

and from the previous property,

each $= \dfrac{la+mc+ne+\ldots\ldots}{lb+md+nf+\ldots\ldots}$

Thus, if $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \ldots\ldots$

then each $= \dfrac{la+mc+ne+\ldots\ldots}{lb+md+nf+\ldots\ldots}$

If is obvious that the numbers $l, m, n, \ldots\ldots$ must be such that $lb+md+nf+\ldots\ldots \neq 0$

Let us take an example and verify whether above property holds.

Let $\dfrac{6}{10} = \dfrac{3}{5} = \dfrac{18}{30} = \dfrac{12}{20}$

Taking $l = 8$, $m = 5$, $n = -2$, $p = \frac{1}{4}$

Consider $\dfrac{8 \times 6 + 5 \times 3 + (-2) \times 18 + \frac{1}{4} \times 12}{8 \times 10 + 5 \times 5 + (-2) \times 30 + \frac{1}{4} \times 20}$

$$= \frac{48+15-36+3}{80+25-60+5}$$

$$= \frac{30}{50} = \frac{3}{5}$$

This ratio is thus seen to be equal to each of the ratios with which we started.

§ 2.9 Solved examples :

1. If $\frac{a}{b} = \frac{c}{d}$ show that $\left(\frac{1}{a}+\frac{1}{d}\right)-\left(\frac{1}{b}+\frac{1}{c}\right)$

$$= \frac{(a-b)(a-c)}{abc}$$

Since $\frac{a}{b} = \frac{c}{d}$ we have $ad = bc$

Now. L. H. S. $= \left(\frac{1}{a}+\frac{1}{d}\right)-\left(\frac{1}{b}+\frac{1}{c}\right)$

$$= \frac{d+a}{ad} - \frac{c+b}{bc}$$

$$= \frac{d+a}{bc} - \frac{c+b}{bc} \quad \text{.........} \because ad = bc$$

$$= \frac{(a+d)-(b+c)}{bc}$$

$$= \frac{a[(a+d)-(b+c)]}{abc}$$

(There is a factor a in the denominator of R. H. S. For this reason we multiply the numerator and the denominator by a)

$$= \frac{a^2+ad-ab-ac}{abc}$$

$$= \frac{a^2-ab+ad-ac}{abc}$$

$$= \frac{a^2-ab+bc-ac}{abc} \quad \text{.........} \because ad = bc$$

$$= \frac{a(a-b)-c(a-b)}{abc}$$

$$= \frac{(a-b)(a-c)}{abc}$$

$$= \text{R. H. S.}$$

2. If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a-b}$ then show that $x+z = y$.

Let $\frac{x}{b+c} = \frac{z}{a-b} = k$

$$\therefore \frac{x+z}{b+c+a-b} = k.$$

So that $\frac{x}{b+c} = \frac{z}{a-b} = \frac{x+z}{c+a} = k.$

But $k = \frac{y}{c+a}$

$$\therefore \frac{x+z}{c+a} = \frac{y}{c+a}$$

From this we get $x+z = y$

3. If $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$ then show that each ratio is either $\frac{1}{2}$ or -1

Since $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$

$$\text{each ratio} = \frac{x+y+z}{y+z+z+x+x+y}$$

$$= \frac{(x+y+z)}{2(x+y+z)}$$

$$= \tfrac{1}{2} \ldots\ldots\ldots (\text{if } x+y+z \neq 0)$$

If $x+y+z = 0$ we have $x = -(y+z)$

So that $\frac{x}{y+z} = \frac{-(y+z)}{(y+z)} = -1$

4. If $\frac{b+c-a}{x} = \frac{c+a-b}{y} = \frac{a+b-c}{z}$

show that $\frac{a}{y+z} = \frac{b}{z+x} = \frac{c}{x+y}$

$$\frac{b+c-a}{x} = \frac{c+a-b}{y} = \frac{a+b-c}{z}$$

$$\text{So, each ratio} = \frac{(b+c-a)+(c+a-b)}{x+y}$$

$$= \frac{2c}{x+y} \ldots\ldots\ldots\ldots\ldots\ldots (\text{I})$$

$$\text{Similarly each ratio} = \frac{(c+a-b)+(a+b-c)}{y+z}$$

$$= \frac{2a}{y+z} \dots\dots\dots\dots\dots\dots (\text{II})$$

and each ratio ⟶ $= \dfrac{(b+c-a)+(a+b-c)}{x+z}$

$$= \frac{2b}{x+z} \dots\dots\dots\dots (\text{III})$$

From (I), (II) and (III) we have,

$$\frac{2c}{x+y} = \frac{2a}{y+z} = \frac{2b}{x+z}$$

After dividing by 2 we have $\dfrac{a}{y+z} = \dfrac{b}{x+z} = \dfrac{c}{x+y}$

Exercise .2.4

1. Fill in the blanks.

(1) $\dfrac{a}{5} = \dfrac{b}{2} = \dfrac{a+3b}{\square} = \dfrac{2a-3b}{\square}$

(2) $\dfrac{x}{7} = \dfrac{y}{2} = \dfrac{z}{3} = \dfrac{2x+5y}{\square} = \dfrac{4y-z}{\square} = \dfrac{\square}{7+4+12}$

(3) $\dfrac{x}{l} = \dfrac{y}{m} = \dfrac{z}{n} = \dfrac{2x+3y-4z}{\square} = \dfrac{\square}{5l-2m+n}$

(4) $\dfrac{x}{b+c} = \dfrac{y}{a-b} = \dfrac{\square}{a+c}$

(5) $\dfrac{x}{m-n} = \dfrac{y}{n-l} = \dfrac{z}{l-m} = \dfrac{x+y}{\square} = \dfrac{x-y-z}{\square}$

2. If $\dfrac{a}{b} = \dfrac{c}{d}$ show that

(1) $\dfrac{a^2+b^2}{a^2-b^2} = \dfrac{ac+bd}{ac-bd}$ (2) $\dfrac{a}{(a+b)(a+c)} = \dfrac{1}{a+b+c+d}$

(3) $\dfrac{a+c}{b+d} = \dfrac{\sqrt{ac}}{\sqrt{bd}}$ (4) $\dfrac{a+3c}{b+3d} = \dfrac{7a-5c}{7b-5d}$

(5) $(a-b)-(c-d) = \dfrac{(a-b)(a-c)}{a}$

3. If $\dfrac{a}{b+c} = \dfrac{b}{c+a} = \dfrac{c}{a+b}$ and $a+b+c \neq 0$, show that each ratio is $\frac{1}{2}$

4. If $\dfrac{x}{a+2b+2c} = \dfrac{y}{b+2c+2a} = \dfrac{z}{c+2a+2b}$, show that each ratio is $\dfrac{x+y-z}{a+b+3c}$

5. If $\dfrac{a}{b+c-a} = \dfrac{b}{c+a-b} = \dfrac{c}{a+b-c}$ and $a+b+c \neq 0$, show that each of the ratios is 1

6. If $\dfrac{y+z}{a} = \dfrac{z+x}{b} = \dfrac{x+y}{c}$, show that

$$\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$$

7. If $\dfrac{ax+by}{x+y} = \dfrac{bx+az}{x+z} = \dfrac{ay+bz}{y+z}$ and $x+y+z \neq 0$, show that each of the ratios is $\dfrac{a+b}{2}$

8. If $\dfrac{y+z-x}{a} = \dfrac{z+x-y}{b} = \dfrac{x+y-z}{c}$ then show that

$$\frac{x+y+z}{a+b+c} = \frac{x^2+y^2+z^2}{az+by+cx}$$

9. If $\dfrac{x}{a+2b+c} = \dfrac{y}{a-c} = \dfrac{z}{a-2b+c}$ show that

$$\frac{x+2y+z}{a} = \frac{x-z}{b} = \frac{x-2y-z}{c}$$

10 If $\dfrac{a}{y+z-x} = \dfrac{b}{z+x-y} = \dfrac{c}{x+y-z}$

Hence show that $\dfrac{x}{b+c} = \dfrac{y}{c+a} = \dfrac{z}{a+b}$

§ 2.10 K - method :

By taking each of given equal ratios to be equal to *k* we can solve practically every example. The method is simply mechanical. However answers to some examples are quickly obtained by this method.

Some examples are given below to help to understand how this method is used.

1. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a}{b} = \frac{c}{d} = \frac{\sqrt{ac}}{\sqrt{bd}}$

Let $\frac{a}{b} = \frac{c}{d} = k$, so that $a = bk$, $c = dk$.

$$\therefore \frac{\sqrt{ac}}{\sqrt{bd}} = \frac{\sqrt{bk.dk}}{\sqrt{bd}} = \sqrt{\frac{bdk^2}{bd}} = \sqrt{k^2} = k.$$

$$\therefore \frac{a}{b} = \frac{c}{d} = \frac{\sqrt{ac}}{\sqrt{bd}}$$

2. If $\frac{a}{b} = \frac{c}{d}$ show that $\left(\frac{1}{a} + \frac{1}{d}\right) - \left(\frac{1}{b} + \frac{1}{c}\right)$

$$= \frac{(a-b)(a-c)}{abc}$$

Let $\frac{a}{b} = \frac{c}{d} = k$ $\quad \therefore a = bk$ and $c = dk$, Now,

L. H. S,

$$= \left(\frac{1}{a} + \frac{1}{d}\right) - \left(\frac{1}{b} + \frac{1}{c}\right)$$

$$= \left(\frac{1}{bk} + \frac{1}{d}\right) - \left(\frac{1}{b} + \frac{1}{dk}\right)$$

$$= \frac{1}{bk} + \frac{1}{d} - \frac{1}{b} - \frac{1}{dk}$$

$$= \frac{1}{b}\left(\frac{1}{k} - 1\right) - \frac{1}{d}\left(\frac{1}{k} - 1\right)$$

$$= \left(\frac{1}{k} - 1\right)\left(\frac{1}{b} - \frac{1}{d}\right)$$

$$= \left(\frac{1-k}{k}\right)\left(\frac{d-b}{bd}\right)$$

$$= \frac{(1-k)(d-b)}{kbd}$$

$$= \frac{(k-1)(b-d)}{kbd}$$

R. H. S.

$$= \frac{(a-b)(a-c)}{abc}$$

$$= \frac{(bk-b)(bk-dk)}{(bk)(b)(dk)}$$

$$= \frac{b(k-1) \quad k(b-d)}{k^2 b^2 d}$$

$$= \frac{bk(k-1)(b-d)}{k^2 b^2 d}$$

$$= \frac{(k-1)(b-d)}{kbd}$$

$\therefore$ L. H. S. = R.H.S.

All those properties of equal ratios which we have earlier shown to hold can also be established by using the 'k- method.'

§ 2.11 Proportion.

Uptil now we studied the properties of equal ratios. We make use of these properties in situations which we come across in every day life. Let us take an example.

A fruit vendor sells 5 guavas for the price of 2 Rs. If we wish to find the price of 20 guavas what reasoning process do we employ ?

20 guavas are 4 times 5 guavas.

We naturally reason out that the price of 20 guavas should be 4 times the price of 5 guavs, that is 8 Rs.

Basically we are saying that the ratios between the two numbers denoting the number of guavas equals the ratio of their repsective prices.

When two - ratios are equal their terms are said to be in proportion.

In the above example $\frac{5}{20} = \frac{2}{8}$, so that, numbers 5, 20, 2 and 8 are in proportion.

So, to say that real numbers a, b, c, d are in proportion is the same as saying $\frac{a}{b} = \frac{c}{d}$.

The numbers *a*, *b*, *c*, *d* are called the terms of the proportion. The numbers *a* and *d* are said to be the end terms and the numbers *b* and *c* are said to be the middle terms. Some times *d* is said to be the fourth term.

In short proportion means equality of two ratios

The middle terms of four numbers in proportion may also be equal. For example, $\frac{4}{6} = \frac{6}{9}$, $\frac{12}{18} = \frac{18}{27}$, $\frac{100}{20} = \frac{20}{4}$ etc.

When the middle terms of four numbers in proportion have the same value, this value is said to be the Geometric Mean of the end terms. In the above examples 6 is the geometric mean of

4 and 9, 18 is the geometric mean of 12 and 27 and 20 is the geometric mean of 100 and 4.

When the numbers a, b, b, c are in proportion, *i. e.* when $\frac{a}{b} = \frac{b}{c}$, b is said to be the geometric mean of a and c. Conversely when it is said that b is the geometric mean of a and c it is implied that the ratio a to b is the same as the ratio b to c.

To summarise,

(1) To say that a, b, c, d are in proportion is the same as saying $\frac{a}{b} = \frac{c}{d}$ or that $ad = bc$.

(2) To say that b is the geometric mean of a and c is the same as saying $\frac{a}{b} = \frac{b}{c}$ or that $b^2 = ac$

§ 2.12 Solved examples :

1. A certain number forms the fourth term of a proportion. If the first three numbers are $6a^2b$, $9a^3$, $8b^3$, find the fourth term.

Let the forurth term be x.

$\therefore$ $6a^2b$, $9a^3$, $8b^3$ and x are in proportion.

$$\therefore \frac{6a^2b}{9a^3} = \frac{8b^3}{x}$$

$$\therefore x \times 6a^2b = 8b^3 \times 9a^3$$

$$\therefore x = \frac{8b^3 \times 9a^3}{6a^2b} = 12ab^2$$

$\therefore$ the fourth term is $12ab^2$.

2. Find the geometric mean of $\frac{36a^3}{b^3}$ and $\frac{b}{9a}$.

Let the geometric mean be x

$$\therefore x^2 = \frac{36a^3}{b^3} \times \frac{b}{9a} = \frac{4a^2}{b^2}$$

$$\therefore x = \frac{2a}{b}$$

$$\therefore \text{geometric mean} = \frac{2a}{b}$$

3. The geometric mean of two numbers is 12. The sum of these numbers is 26. Find the numbers.

Let the two numbers be a and c.

$\therefore\ a + c = 26$............ (I)

Their geometric mean is 12

$\therefore\ a \times c = 12^2 = 144$.........(II)

Now, from equation (I) $c = (26 - a)$

$\therefore$ Substituting the value of c in (II) we get,

$a \times (26 - a) = 144$

$\therefore\ 26a - a^2 = 144$

$\therefore\ a^2 - 26a + 144 = 0$

$\therefore\ (a - 18)(a - 8) = 0$

$\therefore\ a - 18 = 0$ or $a - 8 = 0$

$\therefore\ a = 18$ or $a = 8$

Now, $a \times c = 144$

$\therefore$ if $a = 18$, $18c = 144$ $\therefore\ c = 8$

and if $a = 8$, $8c = 144$ $\therefore\ c = 18$

$\therefore$ the numbers are 18 and 8.

Exercise 2.5

1. Find the fourth proportional to the following.

(1) 2, 6, 7 (2) 4, 12, 5

(3) ab, bc, cd (4) x^2, xy, y^2

2. Fill in the blanks the third proportional in the following

(1) 3, 4, □, 20 (2) 11, 5, □, 45

(3) x^2, xy, □, x^2y^3 (4) xy, yz, □, z^2

3. Find the geometric mean of the following.

(1) 14, 56 (2) 5, 20 (3) 40, 25

(4) $ar, \frac{a}{r}$ (5) $\frac{p+q}{p-q}, \frac{p^2-q^2}{p^2q^2}$

4. Find the value of x if the middle term is the geometric mean of the other two.

(1) 16, 24, x (2) 9, 12, x

(3) $a^2 - b^2,\ a + b,\ x$ (4) $\frac{32a^2}{b^2},\ \frac{8a}{b},\ x$

5, Which number should be subtracted from 11, 15, 21 each, so that the middle number so obtained is the geometric mean of the other two ?

6. Which number should be added to 5, 23 and 77, so that the middle number so obtained is the geometric mean of the other two ?

7. Sum of two numbers is 30 and their geometric mean is 9. Find the numbers.

8. If $(ab + bc)$ is the geometric mean of $(a^2 + b^2)$ and $(b^2 + c^2)$, show that $b^2 = ac$.

9. If $(a - c)$ is the geometric mean of $(a + 2b + c)$ and $(a - 2b + c)$, show that b is the geometric mean of a and c.

Miscellaneous Exercise - 2

1. Fill in the blanks.

(1) The ratio of 65 cm. and 1 m. 4 cm. is ....

(2) The ratio obtained by adding 8 to both the terms of $\frac{17}{22}$ is .... than the original ratio.

(3) The positive real numbers a, b and c are such that $\frac{a + c}{b + c} > \frac{a}{b}$. Therefore, the number a has to be ....than b.

(4) $3x - 5y = 0$ $\therefore\ x : y =$ ....

(5) If $\frac{6p - 5q}{6p + 5q} = \frac{1}{2}$, then $\frac{p}{q} =$ ....

(6) If $\frac{x}{4y} = \frac{8}{3}$, then $\frac{x + 4y}{4y} =$ ....

(7) If $\frac{a}{2b} = \frac{11}{5}$, then $\frac{a - 2b}{2b} =$ ....

(8) $\frac{13}{17} = \frac{39}{51}$ $\therefore\ \frac{13}{17 - 13} = \frac{39}{\cdots\cdots\cdots}$

(9) The fourth proportional to 26, 36 and 13 is ....

(10) The geometric mean of 8 and 72 is ....

2. Answer the following.

(1) If $ad = bc$, state the order relation between $\frac{a}{c}$ and $\frac{b}{d}$

(2) If $\frac{x}{y} = \frac{3}{5}$, find y when $x = 45$

(3) If both terms of a ratio are multiplied by a non-zero number m, in what way will the ratio be affected ?

(4) Find the geometric mean of $(x-y)^3$ and $\frac{(x+y)^2}{(x-y)}$

(5) State greater of the two ratios, one being the ratio of the ages of a father and his son and the other being of their ages five years hence.

3. Study the following solved example.

If $\frac{5x+3y}{5x-3y} = \frac{13}{5}$, find $x : y$.

Method 1

$$\frac{5x+3y}{5x-3y} = \frac{13}{5}$$

$\therefore$ by componendo and dividendo,

$$\frac{(5x+3y)+(5x-3y)}{(5x+3y)-(5x-3y)} = \frac{13+5}{13-5}$$

$$\therefore \frac{10x}{6y} = \frac{18}{8}$$

$$\therefore \frac{x}{y} = \frac{18}{8} \times \frac{6}{10} = \frac{108}{80}$$

$$\therefore x : y = 108 : 80 = 27 : 20.$$

Method 2

$$\frac{5x+3y}{5x-3y} = \frac{13}{5}$$

$$\therefore 5(5x+3y) = 13(5x-3y)$$

$$\therefore 25x + 15y = 65x - 39y$$

$$\therefore 15y + 39y = 65x - 25x$$

$$\therefore 54y = 40x$$

$$\therefore \frac{54}{40} = \frac{x}{y} = \frac{27}{20}.$$

$$\therefore x : y = 27 : 20$$

Solve the following examples using both methods above.

(1) If $\frac{4x+5y}{4x-5y} = \frac{53}{3}$, find $\frac{x}{y}$.

(2) If $\frac{4a^2+7b^2}{4a^2-7b^2} = 8$, find the ratio $a : b$

4. The ratio of the present ages of Sumati and Suneeti is $6 : 5$. The ratio of their ages two years before was $5 : 4$. Find their present ages.

5. The ratio of the present ages of a father and his son is 12 : 5. The father's age at the time of birth of his son was 28. Find their present ages.

6. The ratio of two numbers is 8 : 5. If 8 be added to the greater number and 5 subtracted from the smaller number, the greater number becomes twice the smaller. Find the numbers.

7. An amount of 12800 Rs. was divided among Rama, Shiva and Bhiva as 3 : 5 : 8. Find the amount each of them received.

8. If $\frac{a}{b} = \frac{c}{d}$, show that

(1) $\frac{a^2 + c^2}{ab + cd} = \frac{ab + cd}{b^2 + d^2}$ (2) $\frac{ac}{bd} = \frac{a^2 + c^2}{b^2 + d^2}$

9. If $\frac{2x - 3y}{3z + y} = \frac{z - y}{z - x} = \frac{x + 3z}{2y - 3x}$, show that each rotio is equal to $\frac{x}{y}$.

10. If $\frac{a}{z + x} = \frac{b}{x + y} = \frac{c}{y - z}$, show that $a - b + c = 0$

● ● ● ●

Chapter 3

Percentages

§ 3.1 We know that the notion of ratio helps us to compare magnitudes of two things of the same kind. In our day to day life we are forced to make comparisons on many occasions. For example, let us suppose that Manish secures 483 marks out of 600 marks in an examination and Mitali secures 563 marks out of 800 marks in her examination. If now Mitali claims that her performance in the examination was better than that of Manish, can we say that she is justified in her claim ? To decide whether her claim is right we will have to find the difference between the ratios $\frac{483}{600}$ and $\frac{563}{800}$ and decide which ratio is greater.

In this particular case it would be easy to make a decision if these ratios had the same denominator. For convenience, without changing their values the ratios are so changed that they have the same denominator 100. In this example,

$$\frac{483}{600} = \frac{483}{600} \times \frac{100}{100} = \frac{\frac{483}{600} \times 100}{100} = \frac{80\frac{1}{2}}{100}$$

and $$\frac{563}{800} = \frac{563}{800} \times \frac{100}{100} = \frac{\frac{563}{800} \times 100}{100} = \frac{70\frac{3}{8}}{100}$$

Now we can immediately say the performance of Manish was better than that of Mitali. Percentage is a ratio in which the second term is 100. In the above example Manish obtained $80\frac{1}{2}$ percent marks in the examination, whereas Mitali obtained $70\frac{3}{8}$ percent marks in her examination. Percentage is not a notion which is very much different from the notion of a ratio. At the most we can say that it is a particular type of ratio. It is advantageous always to keep this in mind.

Apart from helping us to compare ratios, percentages have wide applications in calculations of various taxes, commission on objects purchased or sold and the study of growth of population.

The notion of percentage is not novel to you. You have used this notion in various arithmetical problems. Let us now revise our ideas by studying some examples.

§ 3.2 Illustrative examples :

1. The rateable value of a house is 15 percent less than its actual annual rent. Municipality charges 16 percent of the rateable value as property tax and further 8 percent as conservancy tax. If the annual rent of a house is Rs. 6,540, find the amounts of property tax and conservaney tax to be paid annually. How much total tax will have to be paid to the Municipality every six months ?

Rateable value of the house after deducting 15 percent from the annual rent

$$= (\text{ annual rent }) \times \frac{85}{100} = 6540 \times \frac{85}{100} = \text{Rs. } 5559$$

$$\therefore \text{ Property tax} = (\text{rateable value}) \frac{16}{100}$$

$$= 5559 \times \frac{16}{100}$$

$$= \text{Rs } 889.44$$

$$\text{and consenvancy tax} = (\text{rateable value}) \frac{8}{100}$$

$$= 5559 \times \frac{8}{100}$$

$$= \text{Rs. } 444{\cdot}72$$

$$\therefore \text{ total annual tax} = 889{\cdot}44 + 444.72$$

$$= \text{Rs. } 1334{\cdot}16.$$

$$\therefore \begin{array}{c}\text{total tax per}\\ \text{six months}\end{array} = \frac{1334{\cdot}16}{2}$$

$$= \text{Rs } 667{\cdot}08$$

2. In the annual budget of a certain state government, annual income was estimated at 500 crores of rupees and a deficit of 4 crores of rupees was shown. Actually, that year, the annual income increased by 8 percent and the annual expenditure increased by $6\frac{1}{4}$ percent. Determine whether there was actual surplus or deficit in the accounts of that state government, that year.

Estimated income = 500 crores of rupees

Estimated expenditure = (estimated income) + (estimated deficit)

$$= 500 + 4 = 504 \text{ crores of rupees.}$$

$$\text{Actual income} = 500 + \left(500 \times \frac{8}{100}\right) = 500 + 40$$

$$= 540 \text{ crores of rupees.}$$

$$\text{Actual expenditure} = 504 + \left(504 \times \frac{\frac{25}{4}}{100}\right)$$

$$= 504 + \frac{126 \times 25}{100}$$

$$= 504 + 31{\cdot}5$$

$$= 535.5 \textit{ crores of rupees.}$$

$$\therefore \text{ actual surplus} = (543 - 535.5)$$

$$= 4.5 \text{ crores of rupees.}$$

Exercise 3.1

1. A merchant purchased fine cloth worth Rs. 14,750, coarse cloth worth Rs. 8,475 and ready-made clothes worth Rs. 16,960. He had to pay octroi duty at the following rates : 12% on fine cloth, 2% on coarse cloth and 8% on ready-made clothes. How much octroi duty had he to pay ?
2. A shopkeeper marks the selling price of an article 20% above his cost price, but allows 8% discount on the marked price to a customer. What price will a customer have to pay for an article which cost the shopkeeper Rs. 645 ? How much percent is the profit of the shopkeeper ?
3. The population of a town was 12725. 8% of the population left the town to seek employment elsewhere. But due to the establishment of a sugar factory in the town 1527 people from other places came to the town. What was the parcentage increase or decrease in the population of the town ?
4. In 1981 the family of Satbhais had to incur expenditure $12\frac{1}{2}\%$ more than that in 1980, due to increase in the cost of living. In 1982, the expenditure incresed by 8% over that in 1981, What was the percentage increase in 1982 over that in 1980 ? If the monthly expenditure of the family in 1980 was Rs. 840, what was its monthly expenditure in 1982 ?
5. A certain number of workers were employed in a factory at its starting time. After six months the number of workers had to be increased by 16%. Then by the end of the year there was again an increase of 5% in the number of workers. What was the total percentage inrease over the number of workers at the start, by the end of the year ?
6. Shivajirao Patil sold his farm at 20% profit to Dada Shinde, who again sold it back to Shivajirao Patil at 35% profit. By what percentage increase over his cost price did Shivajirao Patil pay to Dada Shinde ? If shivajirao Patil had purchased the farm for Rs. 28,500 originally, for what amount did he purchase it from Dada Shinde again ?

7. Post and Telegraph Department announced a raise of 15% in the cost of Postal envelopes. This resulted in a fall in their sales by 20%. Consequently, there was a decrease of Rs. 104 in the monthly sales of envelopes at a certain Post Office. What were the monthly sales of Postal envelopes at that Post Office prior to the increase in the rate?
8. Railway Ministry declared a 6% raise in the second class fares. This resulted in decreasing the passenger traffic by 5%. Inspite of this decrease, there was an increase of Rs. 1400 in the monthly sales of tickets at a certain Railway Station. What was the monthly income of sales of tickets at the station prior to the increase in the fares?
9. Because of the raise of 20% in the price of sugar, the daily sales of sugar at a certain grocery shop decreased by 14%. Even then, the total daily sales of sugar at the shop increased by Rs. 128. What were the daily sales of sugar in that shop before the increase in the price of sugar?
10. Due to the increase of 25% in the cost of a book, its sales are reduced by 10%. Inspite of it, the monthly sales of the book increased by Rs. 3,650. What were the monthly sales of the book originally?

§ 3.3

We have seen that behind the basic notion of percentages is the idea of a ratio, We have worked out some problems to stress this notion and to have practice in its application. We shall now study some more problems to gain a deeper understanding of the concept of percentages.

Illustrative Example - 1

In ten years, the population of a town increases by 4%. Today its population is 17,576. What was its population 20 years ago?

Let the population of the town 20 years ago be P. So, in ten years it would have increased to

$$P \times \frac{104}{100}$$

In another ten years its population will have increased and come to

$$\left(P \times \frac{104}{100}\right) \times \frac{104}{100} = P \times \left(\frac{104}{100}\right)^2$$

But this population today is 17,576.

$$\therefore \quad P\left(\frac{104}{100}\right)^2 = 17{,}576$$

$$\therefore \quad P = \frac{17{,}576 \times 100 \times 100}{104 \times 104}$$

$$\therefore \quad = 16{,}250$$

$\therefore$ the population of the town 20 years ago was 16,250.

[In examples of this type, we can use the formula used in examples on compound interest, viz. $A = P\left(1 + \frac{r}{100}\right)^n$.

If P = population 20 years ago,

$r = 4$ = rate of increase in population every 10 years,

$n = 2$ = the number of periods (units of 10 years)

A = the present population = 17,576,

then, we have,

$$17{,}576 = P\left(1 + \frac{4}{100}\right)^2,$$

which is the same equation as we have obtained above.]

Exercise 3.2

1. A farmer increases his output of Jowar in his farm every year by 8 %. This year he produced 2,187 quintals of Jowar in his farm. What was the yearly produce of Jowar two years ago ?

2. A piece of machinery was purchased by a factory. In the balance sheet prepared at the end of every year, the value of the machinery was shown after successive depreciation of 10%. At the end of the second year the value of the machinery in the balance sheet was shown at Rs. 12,960. What was the cost price of the machinery ?

3. A person deposited a certain sum of money in a Savings Bank at 12% interest per annum. Instead of withdrawing the interest at the end of a year he allowed it to accumulate it in his account. At the end of the second year he had to his credit an amount of Rs. 12,544 in his account. How much amount had he deposited ?

4. A shopkeeper increases every month his sales by 5% over those of the previous month. His sales in August 1982 were Rs. 5,292. What were his monthly sales in June 1982 ?

Illustrative Example - 2.

In a certain examination 70% of the students passed in Science and 65% in Mathematics. The percentage of students failing in both the subjects was 22. The number of students passing in both the subjects was 684. Find the number of students that appeared at the examination.

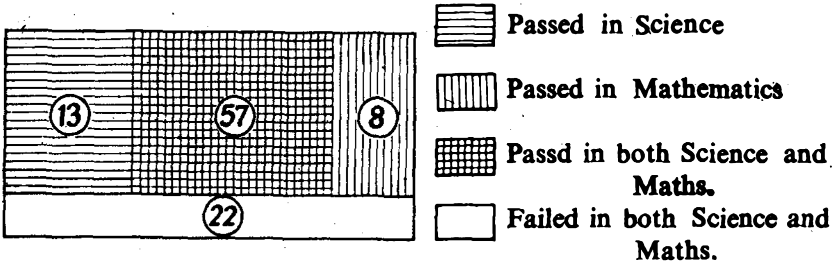

13
57
8
22
Passed in Science
Passed in Mathematics
Passd in both Science and Maths.
Failed in both Science and Maths.

Fig 3.1

Let us assume that 100 students appeared at the examination. Let this number of students be denoted by a rectangle. The number of students who failed in both the subjects is 22 and in the diagram it is represented by the blank strip in the rectangle. The number of students who have passed **at least in one subject,** i. e. the number of students who have passed in Science or in Mathematics or in both is $100 - 22 = 78$. Out of these students, those who have passed in Science are 70. This number is represented by the portion of the diagram shaded horizontally. The number of students who have passed in Mathematics but who have failed in Science is $78 - 70 = 8$. The number of students who have passed in Mathematics is 65. So, the number of students who have passed in Science but failed in Mathematics is $78 - 65 = 13$.

The portion of the diagram which is shaded vertically shows the number of students passing in Mathematics. The number of students who have passed both in Science and Mathematics is covered by the doubly (both horizontally and vertically) shaded portion and it is obviously $78 - (13 + 8) = 78 - 21 = 57$.

This means that 57% of the students passed both in Science and Mathematics and their number is given to be 684.

Now, if x is the number of students who appeared at the examination, then

$$x \times \frac{57}{100} = 684$$

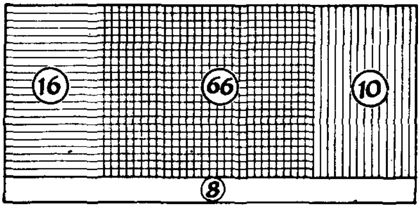
$$\therefore\ x = 684 \times \frac{100}{57} = 1200$$

$\therefore$ 1200 students had appeared at the examination.

Exercise 3.3

1. In a certain town, the percentage of persons speaking Marathi is 82, while the percentage of persons speaking Hindi is 76. The number of persons speaking both Marathi and Hindi is 5,544. If the percentage of persons unable to speak either of the two languages is 8% what is the population of the town?

[Note : Use the following diagram and explanation below :



▤ Speaking Marathi
▥ Speaking Hindi
▦ Speaking both Marathi and Hindi
☐ Unable to speak either Marathi or Hindi.

Fig 3.2

Let x be the population of the town.

Then $x \times \frac{66}{100} = 5{,}544$.]

2. 38% of the students of a school play hockey, 27% of the students play football and the number of students who play both the games is 184. The number of students who neither play hockey nor football is 58. What is the student population of the school ?

3. 1365 students appeared at an examination in two subjects English and Mathematics. 43% of them passed in both the subjects. 65% of the students passed in Mathematics while 58% passed in English. Find the number of students failing in both the subjects.

4. 1820 students appeared at the theory and practical examinations in physics at the graduation level. Percentage of students failing in both the parts was 12. Only 65% of the students passed in theory, while 78% passed in practicals. Degrees were awarded to those who passed in both the parts. Find the number of students who received the degree ?

Illustrative Example - 3

A pharmaceutical dealer has 200 grams of insecticide of 30% concentration. He added to it some quantity of 5% concentrated insecticide and obtained a mixture of 10% concentration. What quantity of 5% concentration did he add ?

While solving such examples we should bear in mind the following two important facts :

(1) (weight of the first solution) + (weight of the second solution) = (weight of the mixture)

(2) (weight of the solute in the first solution) + (weight of the solute in the second solution) = (weight of the solute in the mixture)

Let the quantity of insecticide added of 5% concentration be x grams.

	Solution 1		Solution 2		Mixture
Weight of solution	200 gm		x gm		$(200+x)$ gm
Percentage of solute	30%	+	5%	=	10%
Weight of solute	$200 \times \frac{30}{100}$		$x \times \frac{5}{100}$		$(200+x) \times \frac{10}{100}$

$$\therefore \quad 200 \times \frac{30}{100} + x \times \frac{5}{100} = (200 + x) \times \frac{10}{100}$$

$$\therefore \quad 200 \times 30 + 5x = (200 + x) \times 10$$

$$\therefore \quad 6000 \quad + 5x = 2000 + 10x$$

$$\therefore \quad 4000 = 5x$$

$$\therefore \quad 800 = x$$

$\therefore$ the weight of insecticide of 5% concentration was 800 grams.

Illustrative Example - 4

A dealer mixed some 32% alcoholic solution with some 12% alcoholic solution to obtain 300 grams solution of 20% concentration. Find the quantities of 32% and 12% concentration solutions.

Let the quantity of 32% solution be x grams. The total mixture obtained is 300 gm. So, the quantity of 12% solution is $(300 - x)$ gm.

	Solution 1		Solution 2		Mixture
Weight of solution	x		$(300 - x)$		300
Percentage of alcohol	32%	+	12%	=	20%
Weight of alcohol	$x \times \frac{32}{100}$		$(300 - x) \times \frac{12}{100}$		$300 \times \frac{20}{100}$

$$\therefore \quad x \times \frac{32}{100} + (300 - x) \times \frac{12}{100} = 300 \times \frac{20}{100}$$

$$\therefore \quad 32x + 12\,(300 - x) = 300 \times 20$$

$$\therefore \quad 32x + 3600 - 12x = 6000$$

$$\therefore \quad 20x = 2400$$
$$\therefore \quad x = 120$$

$\therefore$ the 32% solution was 120 gm, and
the 12% solution was $(300 - x) = 300 - 120 = 180$ gm.

Exercise 3.4

1. 80 grams of 5% Boric Acid solution was added to some quantity of 12% Boric Acid solution, to obtain a mixture of 10% concentration. What was the quantity of the 12% solution ?
2. A dairyman wants to mix some 3% fat milk and with some 8% fat milk to obtain 50 litres of 6 % fat milk. Find the quantities of 3% and 8% fat milk he should mix together.
3. Some quantities of alloys containing 30% and 55% of pure silver were melted together to obtain 400 grams of an alloy containing 40% of silver. Find the quantities of the 30% and 55% alloys.
4. In a laboratory there is 25 grams of 8% hydrochloric acid solution. How much 20% hydrochloric solution should be added to it to obtain a 12% solution ?

Miscellaneous Exercise – 3

1. Asgaralli purchased a T. V. set for Rs. 4,500 and sold it to Subhash at a profit of 12%. Subhash then sold it to Suresh at a profit of 5%. How much percent more did Suresh pay than Asgaralli for the set ?
2. 3500 students were sent up by a district for the competition at the state level. 40% of them were successful. These successful students were then sent up for the competition at the national level. 8% of the students sent up were selected at the national level. How many were they ? What percentage of the students sent up from the district were selected at the national level ?
3. In a forest there were 30,000 teakwood trees two years ago. Due to severe cutting of trees, the number of trees reduced every year by 6%. How many teakwood trees are there now in that forest ?

4. Applicants for service in a bank were subjected to an examination in English and Mathematics. 80% of the candidates passed in English and 75% in Mathematics. 18% of the candidates failed in both the subjects. If the number of candidates passing in both the subjects was 219 who were taken up in service, how many applicants were there ?

5. Due to industrialisation, the population of a city went on increasing by 7% every five years. The present population of the city is 1,14,490. What was the population 10 years back ?

6. A 16% increase in the bus fare resulted in a 6 % decrease in passenger traffic. Even then the daily collection of a bus depot increased by Rs. 5,424. Find the daily collection of the depot before the increase in the fare.

7. In 150 litres of 3% fat milk some quantity of 8% fat milk was added to get a mixture of 6% fat milk. How many litres of 8% fat milk was added ?

8. Two alloys containing 30% of chromium and 8% of chromium were melted together to obtain an alloy weighing 250 grams and containing 19% chromium. Find the quantities of the two alloys melted together.

9. The rates of tickets for admission to a drama were increased by 40%. So, the audience was reduced by 25%. Even then, the collection of the theatre for one day increased by Rs. 130. What was the former daily collection of the theatre ?

10. In a certain village, 2460 hectares of land is agricultural land, out of which 57% is for the wheat crop, 52% for Jowar crop and 36% for crops other than wheat and Jowar, during different seasons of a year. How much land comes under the category of producing wheat and Jowar both ?

● ● ● ●

Chapter - 4

Partnership

For starting a sizeable business or for expanding the working of an already existing business, a large capital is needed. Many times one person only cannot provide such a big capital. So, more than one person desirous of making profit, come together and bring in the required capital, to start a business or to expand a running business. The business run by more than one person after bringing in their capitals, is called a Partnership , and the persons involved in it are called its Partners.. They are entitled to a part of the profits, if any, as well as they also have to bear the part of losses that may be incurred.

In our every-day life in our vicinity, we are able to observe various kinds of businesses run in partnrship. We should know how profits or losses are divided among the partners in these partnerships.

In general, the distribution of profits in a partnership business is made at the and of a year. In making this distribution the following main points are observed and provisions to that effect are made in the partnership agreement.

(1) Capital invested in a partnership,
(2) the period for which the capital is invested,
(3) the capacity and knowledge of a partner regarding the business,
(4) the time spent by a partner for the business,
(5) the special efforts made by a partner for the enhancement of the partnership business.

For the present we are going to take into account only two of the above things, viz. the capital invested by a partner and the period for which it is invested, and how the facts determine the distribution of profits. For that purpose, let us understand that—

(1) If the periods for which the capitals of two partners

remain invested are the same, then the profits or losses are distributed in proportion to the capitals invested.

(2) If the capitals invested by two partners are equal, then the profits or losses are distributed in proportion to the periods for which they stand invested.

(3) When the amounts of capitals and also the periods for which they stand invested are both different, then the distribution of profits or losses is made in proportion to the products of the capitals and their respective periods.

Illustrative Example - 1

Shiva and Bhiva invested Rs. 10,000 and Rs. 15,000 respectively in a farm for a year. At the end of the year a profit of Rs. 5,460 was made. Find the share of each in the profit.

Let the shares in the profit of Shiva and Bhiva be Rs. x and Rs. y respectively.

Since the period of investment is the same, the distribution of profit is in proportion of the capitals invested.

$$\therefore \frac{x}{10{,}000} = \frac{y}{15{,}000}$$

$$\therefore \frac{x}{2} = \frac{y}{3} = \frac{x+y}{5} = \frac{5460}{5} = 1092.$$

$\therefore$ Shiva's share in the profit $= x = 1092 \times 2 =$ Rs. 2184,

and Bhiva's share in the profit $= y = 1092 \times 3 =$ Rs. 3296.

Illustrative Example - 2

Ratilal, Shantilal and Manilal started a grocery in partnership by investing Rs, 12,000, Rs. 18,000 and Rs. 24,000 respectively. Find the share of each in the annual profit of Rs. 2,700.

Let the shares in the profit of Ratilal, Shantilal and Manilal be Rs. x, Rs. y and Rs. z respectively.

Periods of investment being same, distribution of profit is in proportion to the investments.

$$\therefore \frac{x}{12{,}000} = \frac{y}{18{,}000} = \frac{z}{24{,}000}$$

$$\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{x+y+z}{9} = \frac{2700}{9} = 300$$

$\therefore$ Ratilal's share $= x = 300 \times 2 =$ Rs. 600,

Shantilal's share $= y = 300 \times 3 =$ Rs. 900, and

Manilal's share $= z = 300 \times 4 =$ Rs. 1,200

Exercise 4.1

1. Abdul and John invested respectively Rs. 450 and Rs. 630 for the seasonal business of crackers. They made a profit of Rs. 228 in the same. Find the share of each in the profit.
2. Ramsing, Basappa and Karim purchased a goods transport truck by investing Rs. 25,000, Rs. 30,000 and Rs. 35,000 respectively and started goods transport business. At the end of a year the the business made a profit of Rs.28,800. Find the share of each in the profit.
3. A, B and C started a business in partnership by investing of 4,000, Rs. 5,000 and Rs. 6,000 respectively. At the end of a year, they found that they have incurred a loss of Rs. 1,350. Find how much loss each one had to bear.
4. Raman, Madan and Arun open a cloth stores after bringing in Rs. 13,500, Rs. 10,500 and Rs. 18,000 respectively. In a year's time they made a profit of Rs. 1,680. Find the share of profit of each one of them.

Illustrative example - 3

Ramanbhai started a stores by investing a certain capital. After 2 months he admitted Vallabhbhai as a partner who invested the same amount. Taking into consideration the need for more capital for the expansion, they invited Chimanbhai into their partnership after 4 months, on condition that he brings in the same amount of capital as they had. At the end of the year when accounts were finalised, they showed a profit of Rs. 8,400. Find how much profit each of the three got.

Let the shares in profit of Ramanbhai, Vallabhbhai and Chimanbhai be Rs. x, Rs. y and Rs. z respectively.

Since the capitals are equal, the distribution of profit is in proportion to the periods of investments.

$$\therefore \frac{x}{12} = \frac{y}{10} = \frac{z}{6}$$

$$\therefore \frac{x}{12} = \frac{y}{10} = \frac{z}{6} = \frac{x+y+z}{28} = \frac{8400}{28}$$

$$\therefore \text{Ramanbhai's share in the profit} = x = \frac{8400}{28} \times 12 = \text{Rs. } 3{,}600$$

$$\text{Vallabhbhai's share in the profit} = y = \frac{8400}{28} \times 10 = \text{Rs. } 3000$$

$$\text{and Chimanbhai's share in the profit} = z = \frac{8400}{28} \times 6 = \text{Rs. } 1800$$

Exercise 4.2

1. A,B and C having brought in,the same amounts started a business in partnership. But after 4 months from the start A withdraw his capital and left. After another 6 months B also left the partnership. At the end of the year, there was a net profit of Rs. .3,900. How is this profit to be divided among A, B and C ?
2. Chandu and Krishna own the same number of cows. They hired a pasture on an annual rent of Rs. 4,800. Chandu grazed his cows for the first four months then Krishna grazed his cows for next five months and again Chandu grazed his cows in the pasture for the remaining period of the year. How much rent should each of them pay ?
3. A, B and C each invested Rs. 16,000 and started a general provision stores. After four months from the start A left and after another five months B left the partnership. At the end of the year the accounts disclosed that there was a loss of Rs. 6,900 in the enterprise. How much loss should each of the three bear ?

4. Sudhakar started a business on an investment of Rs. 2,000. After three months, Madhuker joined him in partnership with a capital of Rs. 2,000. After another three months Anil joined them bringing in Rs. 2,000. At the end of the year the business showed a loss of Rs. 675. So they dissolved the partnership. How much amount after deducting the losses, did each of them get back ?

Illustrative Example - 4

Krishna opened a small-scale industry of manufacturing note books, by investing Rs. 2000. Due to the paucity of capital, he invited Gopal into partnership after 4 months and he brought in Rs. 1,500. At the close of the year they had made a profit of Rs. 1,050. How should they divide the profit between themselves ?

Let the shares in profit of Krishna and Gopal be Rs. x and Rs. y respectively.

Since the amounts as well as the periods of the investments are both different, the distribution of profit will be in proportion to the products of the amounts and periods of the capitals.

$$\therefore \frac{x}{2{,}000 \times 12} = \frac{y}{1{,}500 \times 8}$$

$$\therefore \frac{x}{24{,}000} = \frac{y}{12{,}000}$$

$$\therefore \frac{x}{2} = \frac{y}{1} = \frac{x+y}{3} = \frac{1050}{3} = 350$$

$\therefore$ Krishna's share in the profit $= x = 350 \times 2 =$ Rs. 700, and Gopal's share in the profit $= y = 350 \times 1 =$ Rs. 350.

Exercise 4.3

1. Radhakisan started a cycle-repairing shop investing Rs. 1,600. After two months he admitted Appa pawar into partnership on condition of his investing Rs. 2,000. Accounts were settled after six months and they showed a profit of Rs. 990. Find the share of each of them in the profit.
2. A started a business with a capital of Rs. 2,000. After 5 months B accepted his partnership with a capital of Rs. 1,800.

After 9 months from A's starting the business, C joined the two with a capital of Rs. 1,200. At the close of the year the business realised a profit of Rs. 2,010. How much share the in this profit will C get ?

3. Gautam and Sukhadeo hired a pasture for a year on a rent of Rs. 1,320. Gautam grazed his 18 cattle for 8 months and Sukhadeo grazed his 30 cattle for the remaining period of the year. How much rent should each pay ?
4. Popatlal and Gulabshet started a business in partnership by bringing in Rs. 12,000 and Rs. 15,000 respectively. Gulabshet left the partnership after 6 months. After 2 months since then Rasiklal joined Popatlal in partnership, investing Rs. 10,000. At the end of the year, the accounts showed a net loss of Rs. 8,220. How much loss had Gulabshet to bear ?
5. Abeed and Sayeed started a business in partnership with Rs. 1,700 and 1,500 as their capitals respectively. After 6 months Sayeed left the partnership. So Abeed brought in Rs. 1,100 more in the business. At the end of the year the business realised a profit of Rs. 1,260. What amount of the profit should Abeed get ?

Miscellaneous Exercise 4

1. A and B formed a partnership by investing Rs. 7,500 and Rs. 4,500 respectively. At the end of the year their business showed a profit of Rs. 1,440. What is the share of each of them in the profit ?
2. Alfred, Allabaksha and Amin opened a store in partnership by investing Rs. 5,200, Rs., 6,500 and Rs. 7,800 respectively. At the end of the year they earned a total profit of Rs. 4,485. Find the share of profit of each of them.
3. Mahadu Patil and Anna Gabale hired a truck for a year for transport of goods, on an annual rent of Rs. 21,000. Mahadu Patil transported goods for the first three months and then Anna Gabale took over for the next seven months and then again Mahadu Patil took over for the last two months. How much rent should each of them pay ?

4. Manikchand invested Rs. 5,000 for six months and Gulabchand invested Rs. 9,600 for seven months in a certain business. They incurred a total loss of Rs. 1,458 in the venture. How much of his capital would each get back after bearing the losses ?

5. Three shepherds Mahadu, Sudama and Harba hired a pasture on a rent of Rs. 1,065. Mahadu grazed his 20 sheep for 9 months, Sudama his 24 sheep for 8 months and Harba, his 28 sheep for 7 months. How should they divide the rent among themselves ?

6 In a certain trade, A had invested Rs, 4,000 for 8 months and B had invested some amount for 4 months. There was a total profit of Rs. 1,750 in the trade, out of which A got his share of Rs. 1,000. What was B's capital ?

7. Dadasaheb and Babasaheb started a busainess in partnership after investing Rs. 15,000 and Rs. 24,000 respectively. Out of the profits, Dadasaheb got Rs. 4,500 while Babasaheb had his share of Rs. 4,800. If Dadasaheb had kept his investment for 6 months, how long did Babasaheb's investment remain in the business ?

8. Babuset ran a grocery for 8 months by investing Rs. 20,000. Maganshet had joined him for 6 months in partnership Out of the total profits in the partnership, Babushet got Rs. 2,500 while Maganshet got Rs. 1,500. How much capital had Maganshet brought in ?

9. A and B started a business in partnership investing same amounts. They made a total profit of Rs. 5,600. A got Rs. 3,200 out of the profits for his investment for 8 months. For how long did B invest ?

10. Shri Joshi invested Rs. 6,000 in a certain business for some time and Shri Rathi invested Rs. 5,000 in the same business for the remaining period of the year. At the end of the year shri Joshi received Rs. 1,500 as his share out of the total profit of Rs. 3,250. Find the periods for which each one of them had invested their monies.

● ● ● ●

Chapter 5

Graph

§ 5.1

Take a horizontal line in the plane of paper. Take a point O on the line L and take another point A to the right of O. Let the

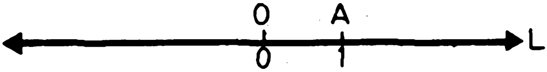


Fig. 5.1

points O and A correspond to the numbers O (zero) and 1 respectively. Let us assume the distance between O and A to be 1. On the line L,· to the right of the point A and at a unit distance from it, take a point B. Take a point C to the right of B and at a unit distance from it.

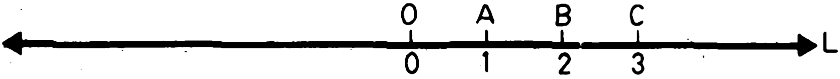


Fig. 5.2

Which numbers will correspond to the points B and C ? To the numbers 2 and 3 respectively.

Similarly on the line L and to the left of the point O, let us take point P, Q, R corresponding to the numbers –1, –2 and –3.

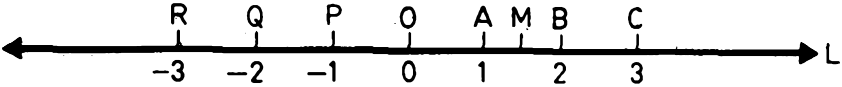


Fig. 5.3

Which number corresponds to the point M the mid-point of seg AB ? $1\frac{1}{2}$ or 1.5.

Points O, A and B of the line L are said to be ' graphs ' of the numbers 0, 1 and 2 respectively and the numbers 0, 1 and 2 are said to be ' coordinates ' of the points O, A and B respectively.

Name the points whose coordinates are $-3, -1$ and 3.

What are the coordinates of the points Q, B and O ?

As the line is indefinite in length we can graph each real number on it. For each real number there is one and only one point on the line L. Moreover, if any point on the line L is taken, then it will correspond to one and only one real number. Such a line L is called a ' number line ' and the point whose coordinate is zero is known as the origin.

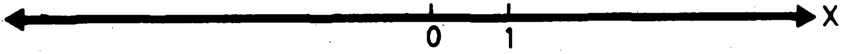


Fig. 5.4

Show the numbers $2, -3, 2{\cdot}5, -0{\cdot}5, 4{\cdot}5$ on the above number line X.

§ 5.2

On any numbers line positive numbers are shown on one side of the number zero and that direction (side) of the line is called the positive direction and the direction opposite to it is called the negative direction.

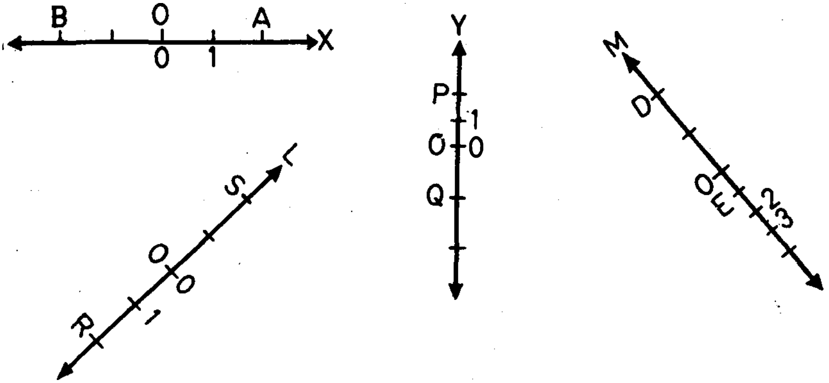


Fig. 5.5

Decide positive direction of each of the above lines.

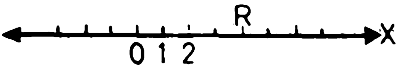
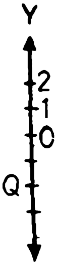


Fig. 5.6



State the coordinate of the point *R* on the number line *X* and the coordinate of the point *Q* on the number line *Y* in fig. 5.6

§ 5.3

Determine the position of the point *P* in the adjoining figure 5.7

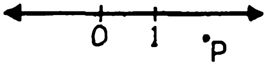


Fig. 5.7

If a point lies on the number line its position can be determined. In fig 5.7 the point *P* is not on the number line, and hence its position cannot be determined by a single number line. Let us see an example which will enable us to determine the position of a point in a plane.

Teacher asked Ahmed, " Where do you sit ? " Ahmed said, " Sir, I sit in the 5^{th} row."

You will easily understand that this information is not sufficient for determining Ahmed's seat.

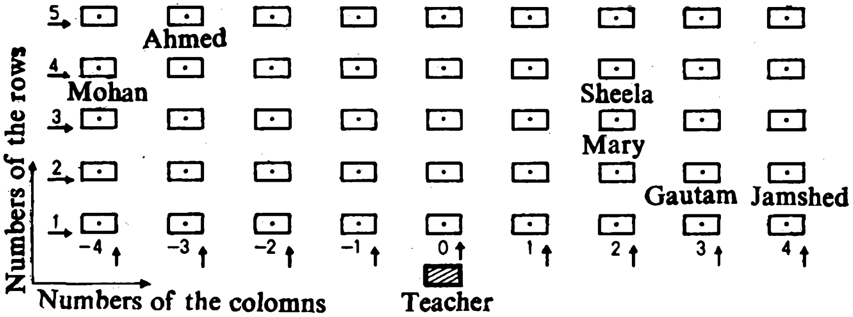


Fig. 5.8

Let us give serial numbers to the rows and columns as shown in the figure. It will enable us to determine seat numbers. Let the columns to the right of the teacher be serially numbered as 1, 2, 3, 4 and to the left as – 1, – 2, – 3, – 4. Number of teacher's column is 0 (zero). Let us number the first row in front of the teacher as 1 and the following successive rows as 2, 3, 4 and 5. While stating seat number first mention the column number and then the row number. Ahmed's column number is -3 and his row number is 5; hence his seat number is $(-3, 5)$

What is Mary's seat number ? Her column number is 2 and row number is 3 ; hence her seat number is (2, 3).

Whose seat number is (3, 2) ?

Note that (2, 3) and (3, 2) represent two different seats. The numbers in such pairs carry different meaning according to the order in which they occur and hence such pairs of numbers are called ' ordered pairs '. The first number in the ordered pair shows the column number and the second shows the row number.

Determine seat numbers of Sheela, Jamshed and Mohan.

In the given diagram, insert the seat numbers (0, 2), $(-2, 1)$ and $(-1, 3)$ in proper places.

The above example illustrates the need of two number lines which are mutually perpendicular (one horizontal and the other vertical) for determining the position of a point in a plane.

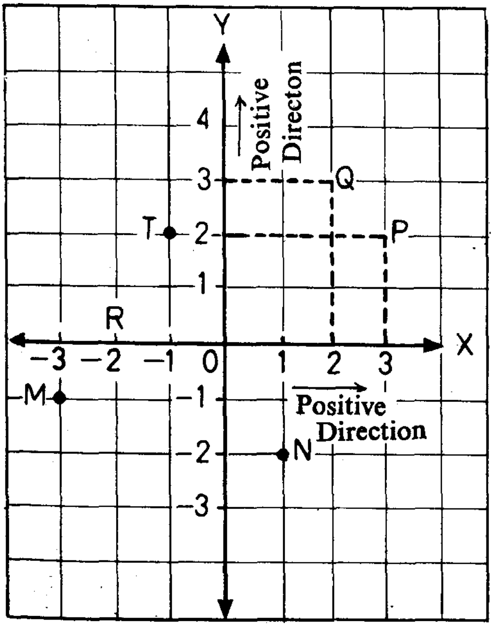


Fig. 5.9

Let us take a horizontal line *X* and a line *Y*, perpendicular to it, as shown in the figure. The point of intersection of these lines is taken as the origin, the direction to the right of the origin is taken as the positive direction of *X* – line (which means that the direction to the left is negative) and the direction above the origin is taken as the positive direction of the *Y* – line (which means that the direction below it is negative). Let us take a point on each line *X* and *Y*, equidistant from the origin and both in the positve direction. Associate these points with the number 1 and the origin with the number 0 (zero). Now the *X* and Y–lines are number lines. On these lines, let us plot the points corresponding to the numbers 2, 3, 4, ... and – 1, – 2, – 3, ... We call these lines as *X* - axis and *Y* - axis. Now can we determine the position of a point in the plane of these axes ? To determine the position of a point *P*, draw perpendiculars from it on both axes. Observe the coordinates of the foot of the perpendicular on both axes. In the figure we can see that the coordinate on the *X*-axis is 3 and on the *Y* – axis is 2. The coordinate of the foot of perpendicular drawn on the number line *X* from the point *P* is the *x*-coordinate of the point *P* and the corresponding coordinate on the number line *Y* (axis) is called its *y*-coordinate. Here *x*-coordinate of the point *P* is 3 and its *y*-coordinate is 2. This can also be said as the point coordinates of the P are (3, 2).

Thus we can say that, two number lines are required for fixing the position of a point in a plane. From the figure 5.9 it is obvious that the coordinates of the point *Q* are (2, 3). The first number 2 in the pair is the *x* – coordinate and the second number 3 is the *y*-coordinate of *Q*. Note that two different points in a plane are represented by the coordinate pairs (2, 3) and (3, 2).

Determine the coordinates of the points *T*, *M* and *N*.

It can be easily seen that the *x*-coordinate of the point *R* on the *X*-axis is – 2. What is its *y*-coordinate ? *X*-axis is perpendicular to the *Y*-axis at the origin. Hence *y*-coordinate of the point *R* will be zero, which means that the coordinates of the point *R* are (– 2, 0). In this way we can determine the position of any point in a plane.

Can we show where the point *A* with coordinates (3, – 2) lie in the plane ? We have to plot the graph of *A* (3, – 2). The *x*- coordinate of the point *A* is 3 and its *y*-coordinate is – 2 ;

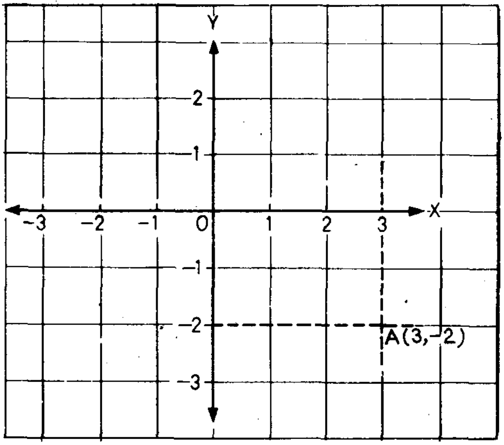


Fig. 5.10

hence draw perpendicular to the X – axis at the point 3 and perpendicular to the Y – axis at the point -2. The point of intersection of these two perpendicular lines is the graph of $A(3, -2)$. In this way we can alway find one and only one point in a plane corresponding to the given pair of real numbers.

From the above discussion we can see that there is a one-to-one correspondence between the points in a plane and the pairs of real numbers formed by two mutually perpendicular number lines. In this way, for determining the position of the points in a plane, a coordinate system can be established.

§ 5.4 Quadrants and coordinates of Points :

The four parts (as shown in the figure) in which a plane is divided by X and Y - axis are known as the quadrants. Starting from the right hand top corner and moving in anticlockwise direction the quadrants which we come across are called as first (I), second (II) third (III) and fourth (IV) quadrants respectively. $P(3, 1)$ is in the first quadrant. The signs of the coordinates

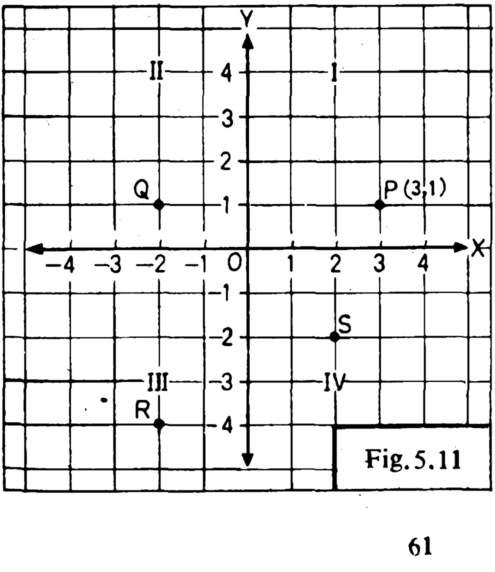


Fig.5.11

of the points Q, R and S characterise the quadrants in which they lie. It is shown in fig. 5.12. In which quadrant will the point whose x – coordinate is negative and y-coordinate is positive lie ? In which quadrant will the point whose x-coordinate is positive any y - coordinate is negative lie ? In which quadrant will the point T $(-4. -3)$ lie ?

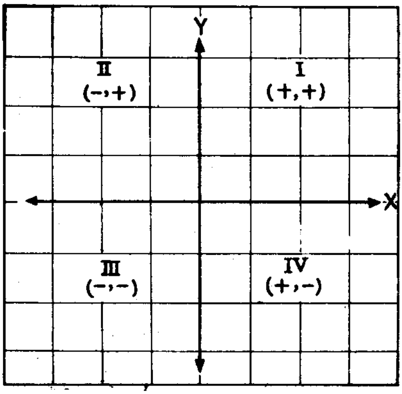


Fig 5.12

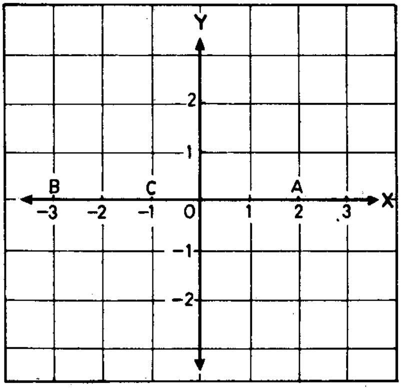


Fig. 5.13

In the figure 5.13, points A, B and C are on the X - axis. Note that the points on the axes do not belong to any quadrant. The x - coordinate of A is 2 and its y - coordinate is 0. What are y - coordinates of the points B and C ?

The y - coordinate of any point on the X - axis is zero.

By considering x- coordinates of few points on the Y - axis, we can easily see that the x - coordinate of any point on the Y - axis is zero.

As the origin lies on both axes, both of its coordinates are zero. Hence the coordinates of the origin are $(0, 0)$.

For determining coordinates of a point in a plane or conversely for plotting the point if the coordinates are given, we have to draw perpendicular lines to both axes. Our labour is considerably saved if we take a paper on which are drawn horizontal and vertical lines at equal distances. Such a paper, having previously drawn horizontal and vertical lines at equal distances, is known as a graph paper.

Exercise 5.1

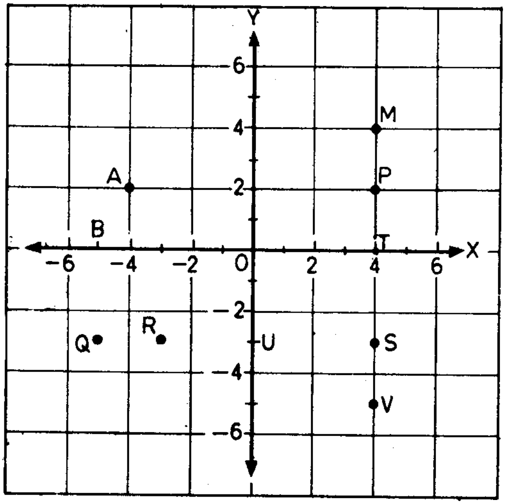


Fig. 5·14

1. (1) State the coordinates of the points T, M Q, P, A, B, U and V.

(2) To which quadrant or to which axis do the above points belong ?

2. Taking 1 cm = 1 unit on both axes, take a coordinate system on the graph paper and plot the following points on it. Write in pencil the coordinates of each point by its side. $A\,(2, 5)$, $B\,(-4, -3)$, $C\,(2, -5)$, $D\,(0, 4)$, $E(-5, 0)$, $F\,(1{\cdot}6, 2{\cdot}8)$, $G\,(-2{\cdot}4, 3)$, $H\,(4, -3{\cdot}2)$, $I\,(-3, 2{\cdot}2)$, $O\,(0, 0)$.

3. State to which quadrant or to which axis do the following points belong ?

$(3, 4)$, $(0, 2)$, $(-3, -5)$, $(-7, 0)$, $(6, -3)$, $(0, -5)$, $(-2, 5)$, $(1{\cdot}5, 2)$, $(\sqrt{2}, -3)$, $(-\sqrt{3}, 0)$, $(1{\cdot}5, -2{\cdot}3)$, $(0, 0)$, $(-\sqrt{3}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{3})$.

4. Complete the following statements.

(1) ——— coordinate on any point on the X-axis is zero.

(2) The point whose x–coordinate is -5 and y-coordinate is 4 lies in the ——— quadrant.

(3) The point $(-2, -5)$ lies in the ——— quadrant.

(4) The point having its x-coordinate zero will lie on the——axis.

(5) The coordinates of the origin are ———

5. Refer fig. 5·14.

(1) What is the x-coordinate of each of the following points M, P, T, S and V? Draw a line passing through each of them.

To which axis is the line parallel ? What will be the x-coordinate of point D (other than the above 5 points) belonging to this line ?

(2) To which axis is the line passing through the points Q, R, U and S parallel ? One coordinate of all points on the line is the same. Which one is it ? What is it ?

§ 5.5 Lines Parallel to the Axes :

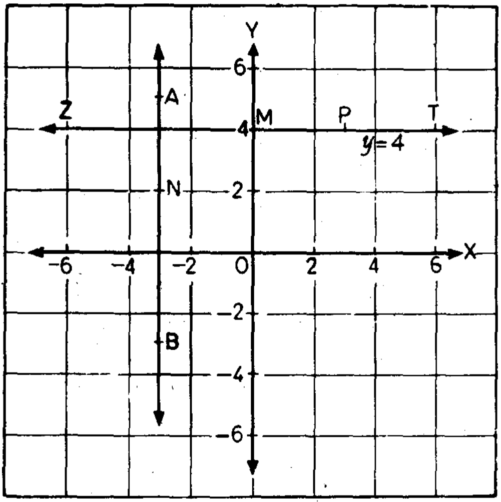


Fig. 5·15

The line PZ in the fig. 5·15 is paralell to the X-axis. Points P (3, 4), M(0, 4), Z (−6, 4) and T(6, 4) are points on it. Observe that y-coordinate of each of these points is 4. If we take any point on this line, its y-coordinate is 4 and any point whose y - coordinate is 4 lies on this line. This is the characteristic of this particular line and therefore it is said that $y = 4$ is the equation of line PZ. In other words we can say that $y = 4$ represents a line which is parallel to X - axis and is at a distance of 4 units above it.

If $y = -3$ is an equation of a line L, then to which axis is L parallel ? To which side of the axis and at what distance from it does it lie ?

Determine the equation of the line AB. To which axis is it parallel ? The equation of line AB is $x = -3$ and it is parallel to the Y - axis and at a distance 3 units towards its left. Line represented by $x = 6$ will be parallel to the Y - axis and at a distance 6 units towards its right. Observe that the equation of any line parallel to the Y - axis is of the form $x = a$. If a is positive the line lies towards right of the Y - axis and if a is negative then it lies towards left of the Y - axis. If a is zero then it will be neither to the left nor to the right of the Y - axis, which means that the line will coincide with the Y- axis. Hence, $x = 0$ is the equation of Y - axis.

Similarly equation of X - axis will be $y = 0$. Recall that the y-coordinate of any point on the X-axis is zero. Equation of any line parallel to the X - axis is of the form $y = b$ where b is a non–zero real number.

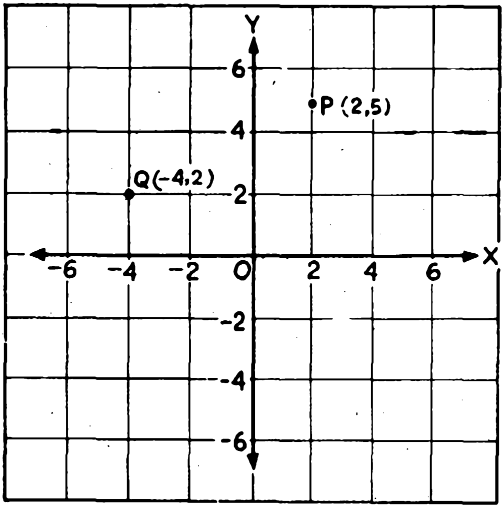


Fig. 5.16

Draw a line parallel to the X- axis through the point $P(2, 5)$. What is its equation ?

$y = 5$.

What is the equation of a line passing through $Q(-4,2)$ and parallel to the Y-axis ?

$x = -4$.

Find the coordinates of the point of intersection of the above two lines. As the point lies on the line $x = -4$, its x- coordinate is -4 and as it lies on the line $y = 5$, its y-coordinate is 5.

Hence the coordinates of the point of intersection of the lines $x = -4$ and $y = 5$, is $(-4, 5)$.

Exercise 5.2

1. Take a coordinate system on the graph paper and plot the following points. $A(4, 5)$, $B(2, 5)$, $C(0, 5)$ and $D(-4, 5)$. Are these points collinear ? If they are collinear draw a line passing through them and write its equation.

2. What is the equation of a line passing through the points $(-3, 5)$ and $(-3, 2)$? To which axis is it parallel ?

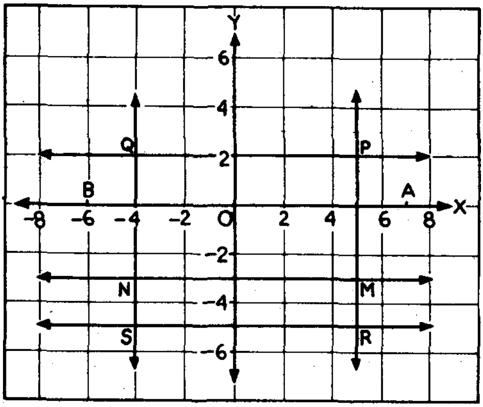


Fig. 5.17

3. Refer the adjoining figure and answer the following questions.

(1) Write the equations of the lines *AB*, *PQ*, *QS* and *RS*.

(2) Which line is represented by the equation $y = -3$?

(3) State the coordinates of the point of intersection of line *MN* and line *PR*.

(4) Write the equation of a line which is parallel to *X*- axis and 5 units below it.

4. Write the equations of the following lines.

(1) Line *AB* is parallel to the *Y*- axis and at a distance 4 units towards its right.

(2) Line *CD* is parallel to the *X*- axis and at a distance 2 units below it.

(3) Line *PQ* is parallel to the *X*- axis and passes through the point $(-3, 5)$.

(4) Line *MN* passes through the point $(-4, 7)$ and is parallel to the *Y*- axis.

5. Write the coordinates of the point of intersection of the lines as pairs of real numbers.

(1) $x = 4, \quad y = -2$ (2) $x = -3, \quad y = 2$

(3) $x = 0, \quad y = -5$ (4) $x = 0, \quad y = 0$

(5) $x = -3, \quad y = 0$ (6) $x = -3, \quad y = -5$

§ 5.6 Graph of Linear Equation in Two Variables :

We have seen that the graphs of the equations of the form $x = a$ or $y = b$ are lines which are either the axes or lines parallel to them. Now let us consider graphs of the linear equations involving two variables x and y.

Consider the equation $y = 2x + 1$. We know that solution of an equation in two variables is not unique. The value of y changes according to the value of x. If we put $x = 1$ in the equation $y = 2x + 1$ we get $y = 2 \times 1 + 1 = 3$. Hence (1, 3) is one of its solution. Let us prepare a table of few solutions.

x	1	0	-2	3	$\frac{1}{2}$
$y = 2x + 1$	3	1	-3	7	2
(x, y)	(1, 3)	(0, 1)	$(-2, -3)$	(3, 7)	$(\frac{1}{2}, 2)$

From the above table we see that (1, 3), (0, 1) $(-2, -3)$, (3, 7), $(\frac{1}{2}, 2)$ are solutions of the equation $y = 2x + 1$. First number in the solution shows the value of x and the second number shows the value of y. We know that pairs of real numbers represent points in a plane and hence the graph of $y = 2x + 1$ will be a set of points in a plane. Plot the above point in your graph copy with a fixed coordinate system. Are these points collinear? Draw a line passing through all of them.

Now let us study graphs of some specific equations.

§ 5.7 Graph of y = x :

Copy and complete the following table on the right hand page of your graph copy.

$y = x$

x	0	1	2	-3	-4		5
$y = x$	0	1				-6	
(x, y)	(0, 0)	(1, 1)		$(-3, -3)$			

With the help of the above table we get some solutions of the equation $y = x$. (see third row of the table.) Take a coordinate system on the left hand page and plot the points corresponding to the above pairs. See figure 5.18. Are the points in the figure *collinear ? Draw the line* passing through them. See figure 5.19. There is an *infinite number of points* on the line which you have drawn. Take one such point *P on it. coordinates of* the point P in the figure 5.19 are (7, 7). Verify that $x = 7$ and $y = 7$ *satisfy* the equation $y = x$. Same thing will be observed for any other point of the line Coordinates of any *point on the line satisfy the* equation $y = x$. Conversely any point correspon*ding to a solution to the* equation $y = x$ will lie on the above line. Hence we can say that the line which is drawn is the graph of the equation $y = x$. The equation $y = x$ and the line which is its graph get identified. The equation identifies the line and conversely the line identifies the equation. Name the line as $y = x$. *The line bisects the first and the third quadrant. Take a point A*

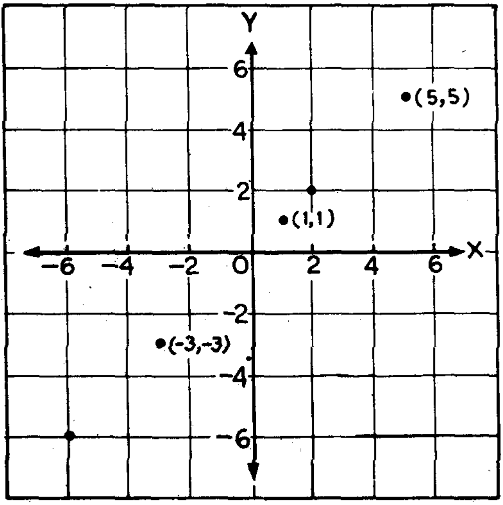


Fig. 5.18

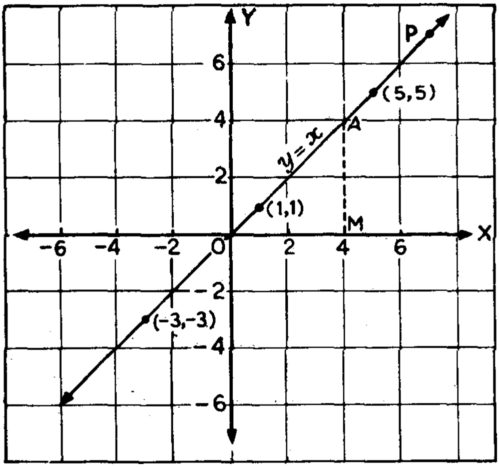


Fig. 5.19

On the graph of $y = x$. Draw seg AM perpendicular to the X–axis. Is the $\triangle OAM$ formed by joining points O the origin, A and M an isosceles triangle ? Why ? What is the measure of the angle formed by the rays OA and OM ? The line which is graph of the equation $y = x$ makes an angle of measure 45° with the X–axis in the first quadrant. In other words this is said as–the line $y = x$ makes an angle of measure 45° with the positive direction of X–axis.

Let us see what is meant by ‘angle made by a line L (which intersects X–axis) with the positive direction of X–axis.’ Take a point P on the line L and in the upper half plane of the X–axis. Let the point of intersection of the line L and the X–axis be Q. On the X–axis, take a point R which is to the right of Q. The angle formed by the ray QP and the ray QR i.e. $\angle PQR$ is the angle made by the line L with the positive direction of X–axis.

§ 5.8 Graph of $y = -x$:

Prepare a table as shown below in your copy and complete it.

$y = -x$

x	4	3	0	-3			$2{\cdot}5$	
$y = -x$	-4		0		5	-7		$-1{\cdot}5$
(x, y)	$(4, -4)$		$(0, 0)$					

Plot the points corresponding to the above solutions on the coordinate system of $y = x$. Are these points collinear ? If so, draw a line passing through all of them. Take a point on this line other than the points which you have plotted and see that the coordinates of the point satisfy the equation $y = -x$. Name the line as $y = -x$. The line which is the graph of equation $y = -x$ makes angle of measure 135° in the second quadrant with the positive direction of X–axis. This line bisects the second and fourth quadrant.

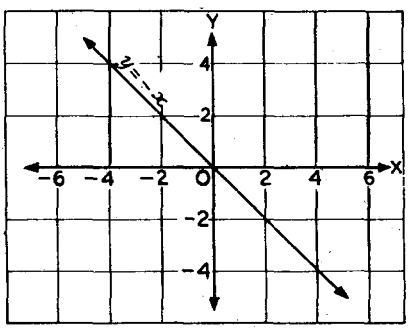


Fig. 5.20

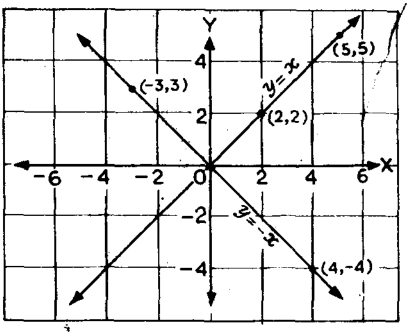


Fig. 5.21

The graph of $y = x$ is shown in fig. 5.19 and of $y = -x$ is shown in fig. 5.20. Both graph are shown together in fig. 5.21.

Following observations can be noted from the graphs of $y = x$ and $y = -x$.

(1) Both graphs are lines passing through the origin.

(2) Graph of the equation $y = x$, except for the point (0, 0) lies wholly in the first and third quadrant whereas the graph of $y = -x$ lies in the second and fourth quadrant.

(3) Both graphs are perpendicular to each other and they bisect the angles between the axes.

§ 5.9 Slope of a Segment and Slope of a Line :

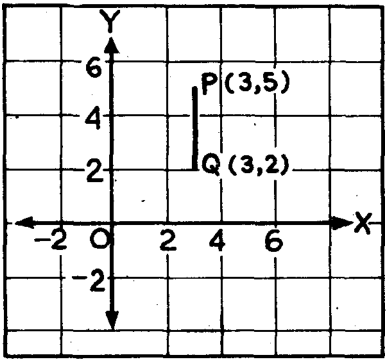


Fig. 5.22 (a)

Observe figure 5.22 (a). The x – coordinate of the point P (3, 5) and Q (3, 2) are same and hence P and Q will lie on the graph of $x = 3$. Seg. PQ is parallel to the Y– axis and perpendicular to the X – axis.

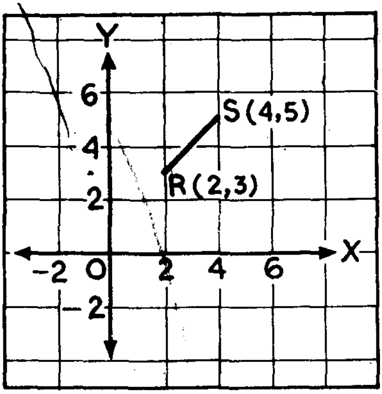


Fig. 5.22 (b)

In the figure 5.22 (b) segment joining the points R (2, 3) and S (4, 5) is not parallel to any axis, it is inclined. Henee we say it has a slope and the segment which is parallel to either axes has on slope.

In Mathematics the word ' slope ' has a specific meaning. See fig. 5.22 (c). Slope of seg. MN is the ratio of the difference between y–coordinates of the points M and N to the corresponding difference between their x– coordinates respectively.

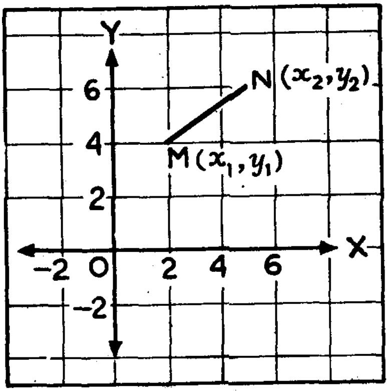


Fig. 5.22 (c)

Slope of seg. $MN = \dfrac{y_1 - y_2}{x_1 - x_2}$, where $x_1 \neq x_2$ which

Sloe of seg. $MN = \dfrac{y_1 - y_2}{x_1 - x_2}$, where $x_1 \neq x_2$ which means that

the x - co–ordinates of the points M and N should be different. If $x_1 = x_2$ then the segment is parallel to the Y - axis. i. e. it is vertical segment has no slope.

Slop of any segment which is non–vertical can be always found.

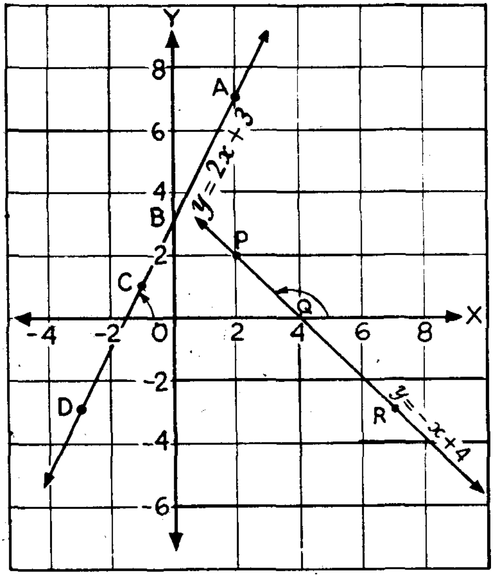


Fig. 5.23

The line AD in fig.5.23 is the graph of $y = 2x+3$. Let us determine the slope of seg. AB of the line AD. If it is denoted by m_1

$$\text{then } m_1 = \frac{(y\text{ - coordinate of } A) - (y\text{ - coordinate of } B)}{(x\text{ - coordinate of } A) - (x\text{ - coordinate of } B)}$$

$$= \frac{7-3}{2-0} = \frac{4}{2} = 2$$

Similarly let us obtain the slope m_2 of seg BD of the line AD.

$$m_2 = \frac{(y\text{–coordinate of } B) - (y\text{–coordinate of } D)}{(x\text{–coordinate of } B) - (x\text{–coordinate of } D)}$$

$$= \frac{3-(-3)}{0-(-3)} = \frac{3+3}{3} = \frac{6}{3} = 2$$

Here $m_1 = m_2$. If we take any two points M and N on the line AD and find the slope of seg. MN, it will be found to be 2. Hence we can say that slopes of any two segments of the same line are equal. Hence slope of a line is the slope of any segment of it. Line PQ in the fig. 5.23 is the graph of $y = -x+4$. Let us determine its slope. Slope of line PQ means the slope of segPQ.

$$\text{Slope of seg } PQ = \frac{(y\text{-coordinate of } P) - (y\text{-coordinate of } Q)}{(x\text{-coordinate of } P) - (x\text{-coordinate of } Q)}$$

$$= \frac{2-0}{2-4} = \frac{2}{-2} = -1$$

Take another two points on the line PQ and determine the slope of segment joining those two points. It will be found to be -1.

Observe figure 5·23. Slope of the line PQ is negative and it makes obtuse angle with the positive direction of the X-axis. Slope of the line AD is positive and it makes acute angle with the positive direction of the X-axis.

To which axis is the line joining points D and R parallel ? (fig. 5·23). What is the slope of line DR ? Hence we can say that slopes of lines parallel to the X-axis, as well as slope of the X-axis is zero. Slope of lines making acute angle with the positive direction of X-axis is positive and the slope of lines making obtuse angle with the positive direction of the X-axis is negative. Lines parallel to Y-axis as well as the Y-axis have no slope.

§ 5.10 Graph of y = mx :

Take a coordinate system in your graph copy and draw the graphs of the following equations on it. (Make table for each equation. Plot atleast 5 different points for each graph.)

$y = \frac{1}{2}x$, $y = x$, $y = 2x$ and $y = 4x$.

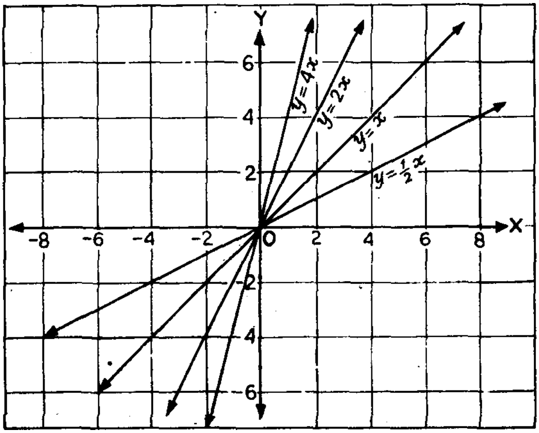


Fig. 5·24

Let us find slope of each line by considering coordinates of two points on it.

slope of $y = \frac{1}{2}x$ is $\dfrac{2-0}{4-0} = \frac{1}{2}$

slope of $y = x$ is $\dfrac{4-2}{4-2} = 1$

slope of $y = 2x$ is $\dfrac{2-0}{1-0} = 2$

slope of $y = 4x$ is $\dfrac{4-0}{1-0} = 4$

What is the relation between the slope of the line and its equation ? All the above equations are of the form $y = mx$, ($m > o$). Slope of each line is the same as the coefficient of x in the corresponding equations. If we observe each graph we find, that equations of the form $y = mx$, ($m > 0$) represent lines passing through the origin and making acute angle with the positive direction of the x – axis. As the value of the slope increases the measure of the corresponding acute angle increases.

Take a coordinate system and draw graphs of the following equations. $y = -4x$, $y = -2x$, $y = -x$ and $y = -\frac{1}{2}x$.

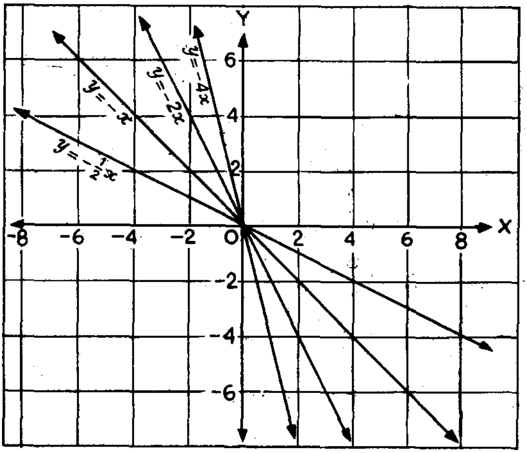


Fig. 5.25

Find slope of each line by considering coordinates of two points of the line.

From the table given below we find that the coefficient of x in the equation of the form $y = mx$, ($m < 0$) gives the slope of the corresponding line.

Equation of a line	Slope of the line
$y = -4x$	-4
$y = -2x$	-2
$y = -x$	-1
$y = -\frac{1}{2}x$	$-\frac{1}{2}$

Each line is making obtuse angle with the positive direction of x – axis. As the value of slope increases (-4, -2, -1, $-\frac{1}{2}$) the measure of the obtuse angle increases. All these lines pass through the origin.

Following observations should be noted in respect of the graphs of the equations of the form $y = mx$.

(1) Graphs of all these equations are lines passing through the origin (0, 0).

(2) Slope of each line is equal to m. If m is positive then the line makes an acute angle and if m is negative the line makes an obtuse angle, with the positive direction of the x– axis.

(3) As the value of m increases from 0 onwards the measure of the acute angle increases from 0° to 90°. When m is negative and it increases towards zero the measure of the obtuse angle increases from 90° to 180°.

Exercise 5.3

1. State giving reason whether the angle made by each of the following lines with the positive direction of X–axis will be acute, right or obtuse.

(1) $y = 2x$ (2) $y = -\frac{1}{3}x$ (3) $3y = x$

(4) $y - 2x = 0$ (5) $2y = 3x$ (6) $x = 5$

2. Which of the following lines do not have slope and which of them have zero slope ?

(1) $x = 5$ (2) $y = -4$ (4) $y = 0$ (5) $x = 0 \cdot y$

3. If the points $(a, 4)$, $(b, 0)$, $(-3, c)$ and $(5, d)$ lie on the graph of the equation $y = -x$, find the values of a, b, c and d.

4. Determine the slope of the segment joining points $P(4, 2)$ and $Q(-8, -4)$. Write the slope of line PQ. Plot the same points P and Q on a graph paper and draw the line PQ. Does it pass through the origin ? What is equation of line PQ ?

5. Refer fig. 5·24 and answer the following questions.

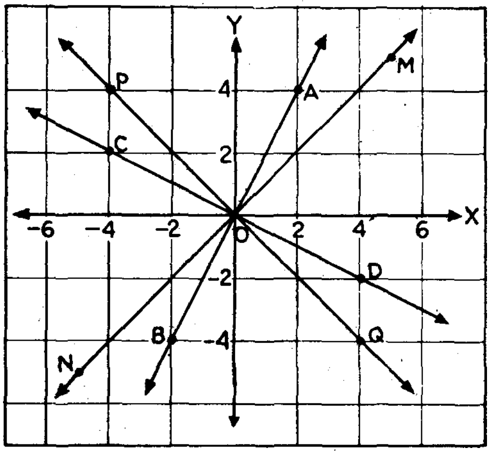


Fig 5·26

(1) Write coordinates of the point M.

(2) Does line MN pass through the origin ?

(3) Find the slope of line MN and hence write its equation.

(4) To which line in the figure, does the point $(-4, 4)$ belong? Write the equation of that line.

(5) Write the equation of line AB.

(6) Find a if the point $(\frac{3}{2}, a)$ lies on the line AB,

(7) To which line do the following points belong ? $(-4, 2)$, $(-2, 1)$, $(2, -1)$, $(6, -3)$. Write its equation.

(8) Write equations of (*a*) line AB (*b*) line CD (*c*) line MN (*d*) line PQ. Order them according to the increasing measure of the angles made by them with the positive direction of the X–axis.

6. Take a coordinate system on the graph paper and draw following graphs on it. (1) $y = 2x$; $y = -\frac{1}{2}x$ (2) $y = -2x$; $y = \frac{1}{2}x$ (3) $y = 3x$; $y = -\frac{1}{3}x$ (4) $y = -3x$. $y = \frac{1}{3}x$. Are the lines in each pair mutually perpendicular ?

We observe that graphs of the equations of the form $y = mx$ and $y = -\frac{1}{m}x$ are mutually perpendicular.

If $m_1 \times m_2 = -1$, the graphs of the equations $y = m_1 x$ and $y = m_2 x$ will be perpendicular to each other.

§ 5.11 Graph of $y = mx + h$:

Draw graphs of the following equations in your copy. Note that each graph is a line. Make tables as shown below and complete them. Determine the slope of each line.

$y = \frac{1}{2}x$

x	0	4	6	−8	−6
$y = \frac{1}{2}x$	0	2			
(x, y)	(0, 0)	(4, 2)			

Slope of $y = \frac{1}{2}x$ is

$$\frac{2-0}{4-0} = \frac{2}{4}$$

$$= \frac{1}{2}$$

$y = \frac{1}{2}x + 3$

x	0	4	6	−8	−6
$y = \frac{1}{2}x + 3$	3	5			
(x, y)	(0, 3)	(4, 5)			

Slope of $y = \frac{1}{2}x + 3$ is

$$\frac{5-3}{4-0} = \frac{2}{4}$$

$$= \frac{1}{2}$$

$y = \frac{1}{2}x - 5$

x	0	4	6	−8	−6
$x = \frac{1}{2}x - 5$	−5	−3			
$(x; y)$	(0,−5)	(4,−3)			

Slope of $y = \frac{1}{2}x - 5$ is

$$= \frac{-5-(-3)}{0-4}$$

$$= \frac{-5+3}{-4}$$

$$= \frac{-2}{-4} = \frac{1}{2}$$

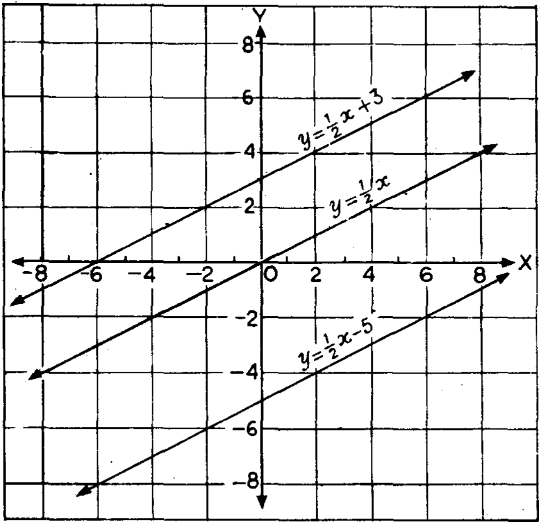


Fig. 5.27

All the above equations are of the form $y = \frac{1}{2}x + h$. Is the slope of each of the above lines the same ? What is the relation between the slope of a line and the coefficient of x in the equation ? Are the graphs of the equations $y = \frac{1}{2}x$, $y = \frac{1}{2}x + 3$ and $y = \frac{1}{2}x - 5$ parallel lines ?

The line corresponding to $y = \frac{1}{2}x + 3$ intersects the y–axis in the point (0, 3). We say that the line makes intercept 3 on the y– axis. In which point does the line $y = \frac{1}{2}x - 5$ intersect the Y-axis ? The line $y = \frac{1}{2}x - 5$ makes an intercept -5 on the Y – axis.

What is the intercept made by the line $y = \frac{1}{2}x + 7$ on the Y–axis ?

We can say that the line $y = \frac{1}{2}x + h$ makes an intercept h on the Y–axis ?

In the equation $y = mx + h$, m denotes the slope of the line and h its intercept on the Y–axis. The equation $y = mx + h$ is known as ' slope – intercept form '.

If two lines have same slope but different intercepts on the Y–axis then they are parallel.

Examples :

1. State the slope and intercept on the *Y*–axis by the line represented by the equation $y = -x + 7$.

The above equation of the line is in the slope – intercept form ($y = mx + h$)

$\therefore$ slope $m = -1$ and intercept made on the *Y*–axis $h = 7$

2. Show that the lines given by the equations $y = x$ and $y = x - 4$ are parallel.

In this example slope of each line is 1 and the intercepts on the *Y*–axis are different (0 and -4). Hence the lines are parallel.

3. Show that the line represented by the equation $y - 3 = x - 5$ passes through the point (5, 3) and is parallel to the line represented by the equation $y = x$.

The line will pass through the point (5, 3) if (5, 3) is a solution of the equation $y - 3 = x - 5$. Substituting $x = 5$ and $y = 3$ in the equation,

$3 - 3 \overset{?}{=} 5 - 5 \,; \quad 0 \overset{\checkmark}{=} 0.$

Hence the line represented by the equation $y - 3 = x - 5$ passes through the point (5, 3).

Let us find its slope.

$y - 3 = x - 5 \quad \therefore \quad y = x - 2$

$\therefore$ slope of the line $= 1$

Slope of both lines is 1 and the intercepts on the *Y*–axis are different. Hence the lines given by the equations $y - 3 = x - 5$ and $y = x$ are parallel.

4. Show that the line $y - 2 = 2 (x - 3)$ passes through the point (3, 2) and is parallel to the line $y = 2x$.

First verify that the line $y - 2 = 2 (x - 3)$ passes through the point (3, 2).

Then show that the slopes of the two lines are same but the intercepts on the *Y*– axis are different.

Hence the line $y - 2 = 2 \, (x - 3)$ passes through the point (3, 2) and is parallel to the line $y = 2x$.

From the above example we can say that the graph of the equation $y - y_1 = m(x - x_1)$ passes through the point (x_1, y_1) and is parallel to the graph of the equation $y = mx$. Or in other words slope of the line $y - y_1 = m(x - x_1)$ is m and it passes through the point (x_1, y_1). The equation $y - y_1 = m(x - x_1)$ is known as ' point–slope form '.

5 Show that the lines $y - 1 = 2(x - 4)$ and $y - 1 = -\frac{1}{2}(x - 4)$ are perpendicular to each other at the point (4_1, 1).

We know that the line $y - y_1 = m(x - x_1)$ passes through the point (x_1, y_1) and is parallel to the line $y = mx$.

∴ The line $y - 1 = 2(x - 4)$ passes through the point (4, 1) and is parallel to the line $y = 2x$.

Similarly the line $y - 1 = -\frac{1}{2}(x - 4)$ passes through the point (4, 1) and is parallel to the line $y = -\frac{1}{2}x$.

Slopes of the lines $y = 2x$ and $y = -\frac{1}{2}x$ are 2 and $-\frac{1}{2}$ respectively. ∴ product of their slopes $= 2 \times (-\frac{1}{2}) = -1$

∴ The lines $y = 2x$ and $y = -\frac{1}{2}x$ are perpendicular to each other.

∴ The lines $y - 1 = 2(x - 4)$ and $y - 1 = -\frac{1}{2}(x - 4)$, which are parallel to the lines $y = 2x$ and $y = -\frac{1}{2}x$, respectively, are perpendicular to each other.

∴ The given two lines are perpendicular to each other at the point (4, 1).

§ 5.12 Intercepts on the Axes :

We say that the line $y = mx+h$ intersects the *Y*–axis in the point $(0, h)$ or the line $y = mx + h$ makes an intercept h on the *Y*–axis. Hence for finding intercept made on the *Y*–axis by any line, we must put $x = 0$ in its equation and find y. (x – coordinate of any point on the *Y*–axis is zero.)

Examples :–

1. Find the intercept made by the line $2y - 3x = 7$ on the *Y*–axis.

As the slope of the line is not required, we need not bring the equation in the slope – intercept form.

Putting $x = 0$ in the equation $2y - 3x = 7$ we get $2y = 7$ $\therefore$ $y = \frac{7}{2}$, which means that the line makes intercept $\frac{7}{2}$ on the $Y-$ axis.

If the intercept made by the line on the $X-$axis is required, we should put $y = 0$ in its equation. Putting $y = 0$ in the equation $2y - 3x = 7$ we get $-3x = 7$ $\therefore$ $x = - {}^{-}\frac{7}{3}$ i. e the intercept made by the line, on the X–axis is $-\frac{7}{3}$ and on the Y–axis is. $\frac{7}{2}$. In other words the line $2y - 3x = 7$ intersects $X-$axis in the point $(-\frac{7}{3}, 0)$ and the $Y-$axis in the point $(0, \frac{7}{2})$.

2. Find the intercepts made by the line $\frac{x}{2} + \frac{y}{3} = 1$ on both axes.

For finding intercept made on the $X-$ axis put $y = 0$ in the equation $\frac{x}{2} + \frac{y}{3} = 1$ $\therefore$ $\frac{x}{2} = 1$ $\therefore$ $x = 2$.

Hence intercept on the $X-$axis is 2,

For finding intercept made on the $Y-$axis put $x = 0$ in the equation $\frac{x}{2} + \frac{y}{3} = 1$ $\therefore$ $\frac{y}{3} = 1$ $\therefore$ $y = 3$

Hence the intercept on the $Y-$axis is 3.

$\therefore$ The line $\frac{x}{2} + \frac{y}{3} = 1$ makes intercept 2 on $X-$ axis and intercept 3 on the $Y-$ axis.

With the help of the above example can you say what intercepts will the line $\frac{x}{a} + \frac{y}{b} = 1$, make on both axes ?

Let us call intercept made on $X-$axis by a line as the X-intercept and on the $Y-$ axis as the $Y-$ intercept.

$X-$ intercept of the line $\frac{x}{a} + \frac{y}{b} = 1$ is a and the $Y-$intercept is b.

The equation $\frac{x}{a} + \frac{y}{b} = 1$ is known as 'two intercepts form.'

3. Write the equation of a line passing through the points $P(2, 3)$ and $Q(4, 7)$ in the points slope form.

We know that the equation of a line passing through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

$\therefore$ The equation of a line passing through the point $(2, 3)$ and having slope m is $y - 3 = m(x - 2)$.

Now let us determine the slope m.

$$\text{Slope of seg } PQ = \frac{(y\text{-coordinate of } P) - (y\text{-coordinate of } Q)}{(x\text{-coordinate of } P) - (x\text{-coordinate of } Q)}$$

$$= \frac{3 - 7}{2 - 4} = \frac{-4}{-2} = 2$$

$\therefore$ The equation of a line passing through the points $P(2, 3)$ and $Q(4, 7)$ is $y - 3 = 2(x - 2)$.

4. Find the equation of a line passing through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Equation of a line passing through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

$$\text{slope of seg } PQ = \frac{y_1 - y_2}{x_1 - x_2}$$

$\therefore$ The equation of a line passing through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $y - y_1 = \dfrac{y_1 - y_2}{x_1 - x_2}(x - x_1)$.

5. Write the equation $2x - 3y + 5 = 0$ in the slope - intercept form and find the slope and the Y-intercept of the line represented by it.

$$2x - 3y + 5 = 0$$

$\therefore$ $-3y = -2x - 5$ (adding $-2x - 5$ to both sides)

$\therefore$ $3y = 2x + 5$ (multiplying both sides by -1)

$\therefore$ $y = \frac{2}{3}x + \frac{5}{3}$. (dividing both sides by 3)

$\therefore$ slope of the line is $\frac{2}{3}$ and the Y-intercept is $\frac{5}{3}$.

6. Find the slope and Y-intercept of the line represented by the equation. $Ax + By + C = 0$

$$Ax + By + C = 0$$

$\therefore$ $By = -Ax - C$

If $B \neq 0$ then $y = -\dfrac{A}{B}x - \dfrac{C}{B}$

Now the equation is in the form $y = mx + h$. Hence the slope $= -\dfrac{A}{B}$ and the Y – intercept $= -\dfrac{C}{B}$.

If $B = 0$. then the equation is $Ax + C = 0$ i.e. $x = -\dfrac{C}{A}$. This means that the line is parallel to the Y–axis.

We know that the lines parallel to the Y–axis as well as the Y–axis have no slope.

$\therefore$ Slope of the line $Ax + By + C = 0$, $(B \neq 0)$ is $-\dfrac{A}{B}$ and its Y–intercept is $-\dfrac{C}{B}$.

From the above example we can say that the equation $Ax + By + C = 0$, $(B \neq 0)$ can be written in the form $y = mx + h$. We know that the graph of the equation $y = mx + h$ is a line. Hence the graph of the general equation of first degree in two variables will be a line. We know that a line is determined by two points. Hence taking two points corresponding to two solutions of the equation $Ax + By + C = 0$. we can draw the line. Let us say that this is the line determined by the equation $Ax+By+C = 0$. Now let us take any point on the line and determine its coordinates from the graph. We can verify that these coordinates satisfy the equation $Ax + By + C = 0$. From this we can say that the equation $Ax + By + C = 0$ represents a line and the equation of the line is $Ax + By + C = 0$.

From the above discussion we can say that Algebra and Geometry are not distinct branches of Mathematics but they have a close relationship between them. The French Mathematician Rene Descartes (1596–1650) was the first person to recognise this relationship. This gave a new lease of life to geometry whose progress otherwise had remained stunted from the time of Greek Civilization.

Exercise 5.4

1. Take a coordinate system and draw graphs of the following equations on it.

(1) $y = x$ (2) $y = x + 4$ (3) $y = x - 3$. Write the slope and Y–intercept of each line. What is the relation between these lines ?

2. Find the equation of a line which is parallel to the line $y = 2x + 3$ and has Y– intercept -4. Draw graph of both equations.

3. Which of the following lines are parallel to $y = -x$? (1) $y = x + 5$ (2) $y = -x + 3$ (3) $x + y = 7$, (4) $2x + 2y = 5$. Verify your answer by drawing graphs.

4. Draw graphs of the following equations with a coordinate system. (1) $y - 2x - 3 = 0$ (2) $2y + x = 5$,

5. Write each of the following equations in the slope intercept form. State slope and Y– intercept of each. (1) $4x - 5y = 3$ (2) $x + y - 5 = 0$ (3) $5y = 7 - x$ (4) $3y = 4$.

6. Find the intercepts made by the following lines on both axes.

(1) $2x + 3y = 6$ (2) $\frac{x}{4} + \frac{y}{3} = 4$ (3) $\frac{x}{2} - \frac{y}{5} = 1$

(4) $2x = 3y - 12$

7. Find a if the point $(a, 5)$ lies on the line $5x + y = 10$.

8. Find A if the point $(4, 3)$ lies on the line $Ax + 2y = 14$.

9. Find the intercepts made by the line $\frac{x}{a} + \frac{y}{b} = 1$ on both axes. Find the slope of the line.

10. Find the slope of the line passing through the points $P(4, 3)$ and $Q(-4, 7)$. Write the equation of the line PQ in the point–slope form.

11. Find h if the point $P(2, 3)$ lies on the line $y = \frac{1}{2}x + h$.

12. State giving reasons whether the lines $y = 3x - 5$ and $y = -\frac{1}{3}x + 7$ are perpendicular to each other. Verify your answer by drawing graphs.

13. Show that the lines $y-2=\frac{2}{3}(x-4)$ and $y-2=\frac{3}{2}(x-4)$ are perpendicular to each other at the point $(4, 2)$.

14. Find the slope of the line passing through the points $A(1, 5)$ and $B(-3, 9)$. Write the equation of the line in the form $y = mx + h$.

15. Are the lines $x+y=5$ and $x-y=1$ perpendicular to each other ? Draw graphs of the above equations. State the coordinates of the point of intersection. Verify that the coordinates of the point of intersection satisfy both equations.

16. Observe fig. 5.28 and answer the following questions.

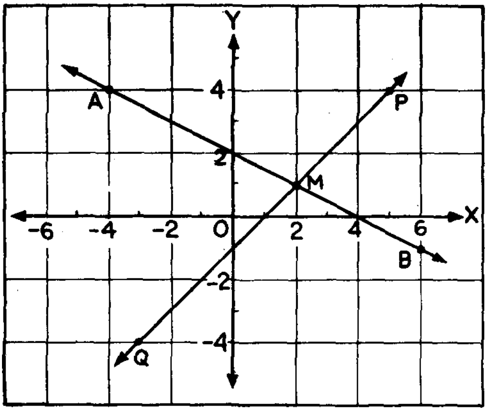


Fig. 5.28

(1) State the Y-intercept and the slope of line AB.

(2) Write the equation of line AB.

(3) Form the equation of line PQ.

(4) Write the coordinates of the point of intersection of line AB and line PQ.

(5) Verify that the coordinates of the point of intersection satisfy both equations.

§ 5.13 Solution of Simultaneous Equations :

We know that the equation $Ax + By + C = 0$ represents a line and the coordinates of every point on the line satisfy the equation $Ax + By + C = 0$

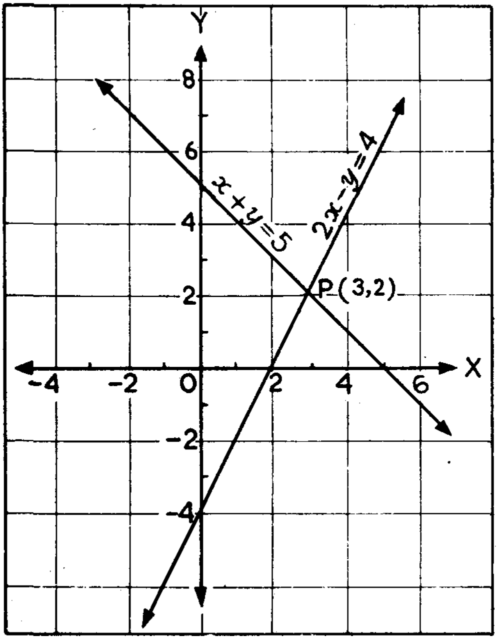


Fig. 5.29

Now let us see how this can be used for solving simultaneous equations in two variables. Study the following example.

Example : Solve the following equations graphically $x+y=5$; $2x-y=4$

First we should draw graphs of both equations. The point P lies on both lines and hence its coordinates will satisfy both equations. The coordinates of the point P are $(3, 2)$. $\therefore$ $(3, 2)$ will be the solution of the equations $x+y=5$ and $2x-y=4$. Verify that $x=3$ and $y=2$. Satisfy both equtiaons.

Thus if want to solve two simultaneous equations then we should take a coordinate system and draw their graphs. The coordinate of the point of intersection of these two lines will be the required solution of these two equations.

Exercise 5.5

Solve graphically the following simultaneous equations.

(1) $x+y=5$; $x-y=1$ (2) $x+y=7$; $x-2y=1$

(3) $y=x-3$; $2x+3y+9=0$ (4) $2x+y=0$; $3x+2y=2$

(5) $x-4=0$; $x+2y-6=0$ (6) $2x-y=3$; $y=2x$

§ 5.14

You must have noticed that the lines which are graphs of the simultancous equations in the last question of exercise 5.5 are parallel lines. You know that the parallel lines do not intersect.

Hence the equations in that last question have no solution. Hence we can say that the simultaneous equations need not necessarily have a solution.

The slopes of the lines given by the equation $2x - y = 3$ and $y = 2x$ are same (each slope being 2) and the Y-intercepts are different (-3 and 0 respectively). Hence if two lines have same slope but have different Y-intercepts then their corresponding equations do not have a solution.

Slope of the line $Ax + By + C = 0$ is $-\frac{A}{B}$ Hence generally for two distinct lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ if $-\frac{A_1}{B_1} = -\frac{A_2}{B_2}$ $\left(\text{i. e. } \frac{A_1}{A_2} = \frac{B_1}{B_2}\right)$ then these two equations do not have any solution.

If two lines have different slopes then the lines cannot be parallel and hence they must intersect in only one point. Hence if $-\frac{A_1}{B_1} \neq -\frac{A_2}{B_2}$ $\left(\text{i. e. } \frac{A_1}{A_2} \neq \frac{B_1}{B_2}\right)$ then the equations $A_1x+B_1y+C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ have one and anly one solution.

Miscellaneous Exercise – 5

1. Take a coordinate system and plot the following points,
(1) P (4, 3) (2) Q (-3, 5) (3) R ($-3{\cdot}5$, 0)
(4) A (-4, -5) (5) B (0, 2·8) (6) C (3, -4)

2. With the help of graph determine whether the following points are collinear. P (2, 5.) Q (0, 3), R (-2, 1). Find the equation of the line PQ and write it in the slope–intercept form.

3, Are the slopes of seg PQ and seg PR in the above question no. 2 the same ? From this information can we say that the points P, Q, and R are collinear ?

4. Which of the following lines are parallel to the line $x + 2y = 3$?
(1) $y = -\frac{1}{2}x + 4$ (2) $2x - y = 1$ (3) $y = \frac{1}{2}x - 3$.

5. Write the equation $3x - 2y = 5$ in (1) Slope intercept form and (2) Two-intercepts form.

6. Write the slope and intercepts on both axes made by the line in question no. 5.

7. State giving reason whether the point P $(4, 3)$ lies on the line $2x - 3y + 1 = 0$.

8. Find the equation of a line passing through the point $P(2, 1)$ and parallel to the line $y = 3x - 5$.

9. Find the slope of a line which is perpendicular to the line $y = 2x - 5$.

10. Find the slope of a line passing through the point A $(0, 2)$ and perpendicular to the line $y - 3x = 5$. Write equation of that line in the point–slope form

11. Which of the following lines pass through the origin ?

(1) $y = 2x$ (2) $y - 3x = 0$ (3) $y - 4 = 2(x - 2)$ (4) $y = 4$ (5) $x = 0$.

12. Find A and B if X–intercept of the line $Ax + By = 6$ is 3 and Y–intercept is 2.

13. Find A and B if slope of the line $Ax + 2y = B$ is -1 and its Y–intercept is 3.

14. Solve graphically the following simultaneous equations. $2x - y = -3$; $2y - x = 3$.

15. Find the intercepts made on both axes by the line $3x -- 5y = 10$.

16. Determine C if the line $5x - 4y = C$ intersects X–axis in the point $(2,0)$. What are the coordinates of the point of intersection with the Y–axis ?

17. Write the equation in the two–intercepts form of the line whose X–intercept is 3 and Y–intercept is -2. Find the slope of the above line.

18. Find the slope of the line which intersects X–axis in the point $(-2, 0)$ and Y–axis in the point $(0, 3)$. Write equation of the line in the slope–intercept form.

19. Find the coordinates of the point of intersection of the lines $2x - y = 5$ and $x = 7$.

20. Find a, if $P(3, a)$ lines on the line $x - y = 5$.

● ● ● ●

Chapter 6

Equations

§ 6.1 Linear Equations : If a, b, c are real numbers and x is a variable, equation $ax+b=c$, $(a \neq o)$ is called a linear equation in one variable. Each equation has a left hand side and a right hand side. Both the sides contain either numerical or algebraic expressions. In case of linear equations index of the varible is never more than one. If the variable is replaced by a number, that number is called the value of that variable. After substituting a number for the variable in the given equation, if both sides of the equation are equal then the equation is said to be satisfied for that value of the variable. If both sides are not equal, the equation is said to be not satisfied for that value of the variable.

The number which satisfies the given equation is called the solution of the equation. To solve an equation means to find its solution.

Now let us consider a set of two linear equations in the same variable These equations are said to be equivalent when a solution of one equation is also the solution of the other and a number which is not a solution of the first is also not the solution of the other.

The equations $x+3=7$ and $x+5=9$ are equivalent equations, because 4 is the solution of both the equations and no other number except 4 is the solution of any of them.

You know that equivalent equations are obtained by performing the following four operations on a linear equation.

(1) by adding the same number on both sides

(2) by subtracting the same number from both sides

(3) by multiplying both sides by the same non-zero number

(4) by dividing both sides by the same non-zero number.

After performing the above operations the final form of the equivalent equation is $x = k$ (k being a real number). The solution of the first equation will be k because it is the solution of latter.

Any given linear equation can be written in the form $ax + b = c$. If we have to solve this equation, we should have to make use of the operations mentioned above and bring the equation in the form $x = \ldots\ldots\ldots$

Subtracting b from both sides

$$ax = c - b$$

Dividing both sides by a

$$x = \frac{c-b}{a}$$

$\therefore\ \dfrac{c-b}{a}$ is the solution of the equation $ax + b = c$.

Examples :

1. $2x - 5 = \frac{3}{7}(x + 3) + 8$

$2x - 5 = \frac{3}{7}(x + 3) + 8$

$\therefore\ 14x - 35 = 3(x + 3) + 56$ (multiplying both sides by 7)

$\therefore\ 14x - 35 = 3x + 9 + 56$

$\therefore\ 14x - 35 = 3x + 65$

$\therefore\ 11x = 100$ (Subtracting $3x$ from both sides)

$\therefore\ x = \frac{100}{11}$ (dividing both sides by 11)

$\therefore\ \frac{100}{11}$ is the solution of the given equation

Check :

Putting $x = \frac{100}{11}$ in original equation

$$2 \times \frac{100}{11} - 5 \overset{?}{=} \frac{3}{7}\left(\frac{100}{11} + 3\right) + 8$$

$$\frac{200 - 55}{11} \overset{?}{=} \frac{3}{7} \times \frac{133}{11} + 8$$

$$\frac{145}{11} \overset{?}{=} \frac{399}{77} + 8$$

$$\frac{145}{11} \overset{?}{=} \frac{1015}{77}$$

$$\frac{145}{11} \overset{?}{=} \frac{145 \times 7}{11 \times 7}$$

$$\frac{145}{11} \overset{\checkmark}{=} \frac{145}{11}$$

Thus the solution set is $\left\{ \frac{100}{11} \right\}$

2. Find the solution of

$$9\,(4x+5) - 7\,(3x+2) = 6\,(x+7) - 5\,(x+5)$$

$$9\,(4x+5) - 7\,(3x+2) = 6\,(x+7) - 5\,(x+5)$$

$\therefore \quad 36x + 45 - 21x - 14 = 6x + 42 - 5x - 25$

(Opening the brackets)

$\therefore \; 15x + 31 = x + 17$ (Colleeting the like terms)

$\therefore \quad 14x = -14$ (Subtracting x and 31 from both sides)

$\therefore \quad x = -1$ (Dividing both sides by 14)

Check :

Substituting $x = -1$ in the given equation,

L. H. S. $= 9(-4+5) - 7(-3+2)$	R. H. S $= 6(-1+7) - 5(-1+5)$
$= 9(1) \; -7(-1)$	$= 6(6) - 5(4)$
$= 9 + 7$	$= 36 - 20$
$= 16$	$= 16$

$\therefore$ L. H. S $=$ R. H, S.

Thus the solution set is $\{-1\}$

3. Solve $\quad \frac{x+12}{3} - \frac{x+4}{5} = \frac{5x+6}{6} - \frac{x+2}{4}$

$$\frac{x+12}{3} - \frac{x+4}{5} = \frac{5x+6}{6} - \frac{x+2}{4}$$

In order to get rid of the denominators let us multiply both sides by 60 (L. C. M. of 3, 5, 6, 4)

$$\frac{60\,(x+12)}{3} - \frac{60\,(x+4)}{5} = \frac{60\,(5x+6)}{6} - \frac{60\,(x+2)}{4}$$

$\therefore \quad 20\,(x+12) - 12\,(x+4) = 10\,(5x+6) - 15\,(x+2)$

$\therefore \quad 20x + 240 - 12x - 48 = 50x + 60 - 15x - 30$

$\therefore \quad 8x + 192 = 35x + 30$ (taking like terms together)

$\therefore \quad -27x = -162$ (subtracting 35 x and 192 from both sides)

$\therefore \quad x = \frac{-162}{-27}$ (dividing both sides by -27)

$\therefore \quad x = 6$

$\therefore$ Solution set $= \{ 6 \}$

Check :

Substituting $x = 6$,

L. H. S. $= \frac{6+12}{3} - \frac{6+4}{5}$

$= 6 - 2$

$= 4$

R. H. S. $= \frac{5 \times 6 + 6}{6} - \frac{6+2}{4}$

$= \frac{36}{6} - \frac{8}{4}$

$= 6 - 2 = 4$

$\therefore$ L. H. S. = R. H. S.

Exercise 6.1

Find the solution sets of the following equations.

(1) $7(x+2) - 2(x-3) = 40$

(2) $(27 + 5y) + 12 = 18y$

(3) $2(x+9) - 5x = 5(2+x)$

(4) $7(m+4) + 5m = 14m$

(5) $2(x+1) - (x-3) = 4(x+5)$

(6) $7(x+5) + 6(2x+1) = 9(x+3) + 4(x+5)$

(7) $9(4y+5) - 7(3y+2) = 6(y+7) - 5(y+5)$

(8) $17(m+4) + 8(m+6) = 11(m+5) + 15(m+3)$

(9) $21(2p+1) - 20(3p+2) = 18(p+2) - 5(6p+11)$

(10) $10a + 15(a-3) = 205$

(11) $5x + (3x+4) - 2 = 7 + (7x-4)$

(12) $2x - 3(3x-5) = 2(x-6) - 9$

(13) $\frac{y+2}{5} + \frac{y+1}{3} = 5$

(14) $\frac{4p+1}{6} - \frac{2p-4}{3} = -\frac{3p}{4}$

(15) $\frac{2m-1}{7} + \frac{m+5}{3} = 4$

(16) $\frac{x-1}{3} + \frac{x+4}{7} = \frac{x}{2}$

(17) $\frac{3x-2}{5}+\frac{4x-1}{7}=\frac{10x}{9}$

(18) $\frac{y+5}{6}-\frac{y+6}{5}=\frac{y+7}{4}-\frac{y+8}{3}$

(19) $\frac{x+4}{8}-\frac{x+3}{5}=\frac{x+2}{4}-\frac{x+1}{6}$

(20) $\frac{x-4}{4}+\frac{3x}{2}=5-\frac{x-2}{4}$

§ 6.2 Equations involving absolute values :

We know the meaning of absolute value of a real number. If x is a non zero real number, x is either positive or negative. If (x) is positive then $(-x)$ is negative and if (x) is negative $(-x)$ is positive. Of the two numbers x and $(-x)$, one is necessarily positive. We represent the positive of the two numbers by $|x|$. $|x|$ is the absolute value of x.

If $x = 0$, $|x| = 0$

If x is positive, $|x| = x$

If x is negative, $|x| = -x$.

Find out the values of

$|-7|$, $|-\sqrt{2}|$, $|5|$, $|\sqrt{2}|$, $|7|$, $|-5|$

The absolute values of a number and its opposite number are the same. $\therefore$ $|x| = |-x|$

Let us solve some examples using the meaning of absolute value.

Examples :

1. If $|x| = 5$, find the value of x.

We know that $|5| = 5$ and $|-5| = 5$

$\therefore$ If $|x| = 5$ then $x = 5$ or $x = -5$.

2. $|y+3| = 6$. Find the value of y.

Say $y+3 = p$ then $|p| = 6$

$\therefore$ $p = 6$ or $p = -6$

$\therefore$ $y+3 = 6$ or $y+3 = -6$

$\therefore$ $y = 3$ or $y = -9$

3. Solve : $|5x+9| = 31$

Say, $|5x+9| = t$ then $|t| = 31$

$\therefore\ t = 31$ or $t = -31$

$\therefore\ 5x+9 = 31$ or $5x+9 = -31$

$\therefore\ 5x = 31-9$ or $5x = -31-9$

$\therefore\ 5x = 22$ or $5x = -40$

$\therefore\ x = \frac{22}{5}$ or $x = -8$

4. Solve : $|y-4| = -10$

Absolute value of a numbes can be positive or zero, but can never be negative. Hence this equation has no solution.

Exercise 6.2

Solve the following equations :

(1) $|2x-4| = 10$ (2) $|4y+3| = 12$

(3) $|x-\frac{3}{7}| = \frac{4}{7}$ (4) $|7x-\frac{21}{25}| = 0$

(5) $5+|y| = 10$

§ 6.3 Linear equations in two variables :

If $a, b, c,$ are real numbers ($a \neq 0, b \neq 0$) then $ax+by = c$ is an equation in two variables. The equation is said to be satisfied for those values of x and y respectively, for which both sides of the equation assume the same value. These values are called the solution of the equation. For example, for the values $x = 2$ and $y = 3$ the equation $5x+3y = 19$ is satisfied. Therefore the pair (2, 3) is a solution of the equation. In the same equation $5x+3y = 19$ if we now take $x = 4$, then we get $20+3y = 19$, therefore $y = -\frac{1}{3}$. So its solution would be $(4, -\frac{1}{3})$. Thus infinite number of solutions can be found for this equation

When there is just one linear equation containing two variables then an infinite number of its solutions can be found. An equation in two variables does not have a unique solution.

Now let us consider the two equations $3x+5y = 7$ and $5x+3y = 9$ together. When we consider two linear equations of two variables, such equations are called simultaneous equations. Each equation has infinite number of solutions. In case there is a pair of numbers which satisfies both the equations, this pair is said to be the solution of the simultaneous equations.

It is not necessary that both the given equations must have a solution. If the given simultaneous equations have a solution then they are said to be consistent. If they do not have a solution, they are said to be inconsistent equations.

Consider the following simultaneous equations,

$x + y = 6$ ……… (I) $2x + 2y = 9$ ………… (II)

From (I), $y = 6 - x$. Substituting this value of y in (II) we get,

$$2x + 2(6 - x) = 9$$

$$\therefore\ 2x + 12 - 2x = 9$$

$$\therefore\ 12 = 9,$$ Which is a contradiction.

Because of this contradiction these equations are inconsistent.

How should we find the solution of the given simultaneous equations ? We can prepare a table of solutions corresponding to each equation. To find the solution of the simultaneous equations we will have to search a solution which appears in both tables. This may not be so easy. We will, therefore, adopt the same method which we made use of when trying to find a solution of an equation in one variable. For this, study the points raised below.

It is obvious that if two given equations are replaced by equivalent equations the new pair of equations will be equivalent to the first pair. Any solution of the new pair of simultaneous equations will also be a solution of the first pair of simultaneous equations. Any pair of numbers which is not a solution of the new pair of simultaneous equations will not be a solution of the first pair of simultaneous equations.

The simultaneous equations which are obtained,

(1) by adding the same real number on both sides,

(2) by subtracting the same real number from both sides,

(3) by multiplying both sides by the same non-zero real number,

(4) by dividing both sides by the same non-zero real number,

are equivalent to the given equations.

In addition to above,

(5) The equations obtained by adding the two equations or by subtracting the second from the first, together with any one of the original equations. and

(6) the equations obtained by adding and subtracting the original equations are together equivalent to the given simultaneous equations

§ 6.4 Solution of simultaneous equations.

You already know to solve the simultaneous equations by using the method of elimination of a variable. Using this method we reduce the two equations to one equation containing just one variable. The method of elimination consists in obtaining finally an equation in one variable by performing suitable operations on the two stmultaneous equations. In this final form the variable which is absent is said to have been eliminated.

Study the followig equations,

$x - y = 5 \ldots\ldots\ldots (\mathrm{I}) \qquad x + y = 15 \ldots\ldots\ldots (\mathrm{II})$

If we add these equations, y gets eliminated and if we subtract first from the second, x gets eliminated.,

Adding (I) and (II),

$2x = 20 \quad \therefore \quad x = 10$

Subtracting (I) from (II),

$2y = 10 \quad \therefore \quad y = 5$

$\therefore$ the solution of given equations is (10, 5)

In this example it was easy to eliminate the variables because the coefficients or the absolute values of the coefficients of the variables in both the equations were the same.

Thus if the coefficients or absolute values of the coefficients of the same variable in both the equations are the same, suitable variable can be eliminated by simple addition or subtraction. But when the coefficients of the same variable 'x' in both the equations are not the same, we have to perform suitable operations on these equations so that we obtain new equations equivalent to the original but the coefficients or the absolute values of the coefficients of the same variable 'x' in the new equations become the same. This will help us to eliminate the variable x.

Study the following examples.

1. Solve : $5x + 7y = 31$........(I), $2x - y = 1$........(II)

Here multiply second equation by 7.

$\therefore$ $5x + 7y = 31$........(III) $14x - 7y = 7$........(IV)

Adding (III) and (IV),

$19x = 38$ $\therefore$ $x = 2$.

Substituting $x = 2$ in (II)

$2 \times 2 - y = 1$ $\therefore$ $y = 3$

$\therefore$ the solution of (I) and (II) is (2, 3).

Check :

Substituting $x = 2$ and $y = 3$ in the given equations

(I) $5 \times 2 + 7 \times 3 \overset{?}{=} 31$

$31 \overset{\checkmark}{=} 31$

(II) $2 \times 2 - 3 \overset{?}{=} 1$

$1 \overset{\checkmark}{=} 1$

Thus the solution set is $\{(2, 3)\}$

2. Solve the following simultaneous equations.

$3(x - 3) - 2(y - 2) = 30$........(I)

$4(x - 5) - 5(y + 3) = 0$........(II)

Simplifying both the equations,

$3x - 2y = 35$........(III)

$4x - 5y = 35$ ....(IV)

Multiplying equation (III) by 4 and (IV) by 3.

$12x - 8y = 140$........(V)

$12x - 15y = 105$........(VI)

Subtracting equation (VI) from (V).

$7y = 35$ $\therefore$ $y = 5$

Substituting $y = 5$ in equation (III)

$3x - 10 = 35$ $\therefore$ $x = 15$

Check :

Substituting $x = 15$ and $y = 5$ in the given equations,

(I) $3(15-3)-2(5-2 \overset{?}{=} 30$ | (II) $4(15-5)-5(5+3) \overset{?}{=} 0$

$3(12)-2(3) \overset{\checkmark}{=} 30$ | $4(10)-5(8) \overset{\checkmark}{=} 0$

$\therefore$ the solution set is $\{(15, 5)\}$.

Exercise 6.3

Solve the follwing simultaneous equations by eliminating either x or y.

(1) $4x+3y = 17$; $3x+4y = 18$

(2) $5x+2y = 16$; $3x+7y = 27$

(3) $3x-17 = -2y$; $3y = 12 - 4x$

(4) $2x = 1+5y$; $7x+3y = 24$

(5) $3x = 15+12y$; $2x-9y = 11$

(6) $4x+y = 34$; $x+4y = 16$

(7) $4x+3y = 43$; $3x-2y = 11$

(8) $47 = 7x+2y$; $5x-4y = 1$

(9) $x+5y = 20$; $x+6y = 22$

(10) $5x+3y = 21$; $2x-y = 4$

(11) $2x-3y = 0$; $2x+3y = 60$

(12) $5x+15 = 3y$; $2x+2 = y$

(13) $7x-10y = 6$; $2x = 1+3y$

(14) $10x-3y = 22$; $5x = 2+3y$

(15) $y+17 = 10x$; $x+9y = 29$

(16) $5(x+2)+3(y-4) = -15$; $7(5-x)+2(2-y) = 0$

(17) $3(x+3)-4 = 2(x+y)$; $3(2y-3) = 4(2+y-x)+3$

(18) $5(x+2y)-(3x+11y) = 14$; $7x-9y-3(x-4y) = 38$

(19) $4(x-2y)-(5x+3y) = 30$; $3(3x+7y)-2(x+9y) = 12$

(20) $16(11x-5y) = 11(3x+y)$; $8x = 1+5y$

(21) $5(x+3) = 3y$; $2(x+1) = y$

(22) $(x+1)+2y = 30$; $(y+3)+4x = 56$

(23) $2(x+7)-6(y-4) = 48$; $2(x+4)+6(y+3) = 60$

(24) $4(x-2) = 5(1-y)$; $3y+4+2x = -24x$

(25) $(x-y)-(2x+1) = -8$; $5(3x-2y)-3(2y-1) = 15$

§ 6.5 Solution of simultaneous equations second method :

The example solved in article 6.4 can be solved by another method as follows. Study it and try to understand the method.

$5x + 7y = 31 \ldots\ldots\ldots (\text{I}) \qquad 2x - y = 1 \ldots\ldots\ldots (\text{II})$.

From equation (II) we have. $y = 2x - 1$, which gives us $2x - 1$ as the value of y. Substituting this value in equation (I) we get,

$5x + 7(2x - 1) = 31$

Which is an equation in one variable, Let us solve this.

$5x + 14x - 7 = 31$

$\therefore \quad 19x = 38 \qquad \therefore \quad x = 2$

Substituting the value of x in equation (II) we get,

$2(2) - y = 1, \qquad \therefore \quad y = 3$

$\therefore$ the solution of the given equation is (2, 3).

Exercise 6.4

Solve the following simultaneous equations by expressing x in terms of y or y in terms of x

(1) $5x + 3y = 13 \quad ; \quad y + 4x = 9$

(2) $3x - 4y = 7 \quad ; \quad 2x + y = 12$

(3) $4x - 3y - 23 = 0 \quad ; \quad x + 2y - 3 = 0$

(4) $3x - 2y = 7 \quad ; \quad 4x - y = 11$

(5) $2x + y = -3 \quad ; \quad 3x - 2y = 13$

(6) $4x - y = 9 \quad ; \quad 3x - 2y = 5$

(7) $2x - 3y = 14 \quad ; \quad 5x + 4y = 12$

(8) $4x + 3y = 20 \quad ; \quad 5x + y = 14$

(9) $3x + 2y = 12 \quad ; \quad 4x + y = 11$

(10) $5x + 4y = 2 \quad ; \quad y + 3x = 4$

§ 6.6 Equations involving fractions :

Examples :

1. Solve : $\frac{2x}{5} + y = 6 \ldots\ldots\ldots (\text{I}) \qquad x + \frac{3y}{4} = 8 \ldots\ldots\ldots (\text{II})$

Let us eliminate the denominators first. So multiply the first equation by 5 and second by 4. We then have,

$2x + 5y = 30 \ldots\ldots\ldots (\text{III})$ $\qquad$ $4x + 3y = 32 \ldots\ldots\ldots (\text{IV})$

It is easier to eliminate x from equation (III) and (IV). So multiply equation (III) by 2 and subtract this new equation from (IV)

$$\begin{array}{r} 4x + 10y = 60 \\ 4x + 3y = 32 \\ \underline{- \quad - \qquad -} \\ 7y = 28 \end{array} \qquad \therefore y = 4$$

Substituting the value of y in equation (IV) we get,

$4x + 12 = 32.$ $\qquad$ $\therefore x = 5$

Check :

Substituting $x = 5$ and $y = 4$ in the given equations we get,

(I) $\frac{2 \times 5}{5} + 4 \overset{?}{=} 6$ $\qquad$ (II) $5 + \frac{3 \times 4}{4} \overset{?}{=} 8$

$6 \overset{\checkmark}{=} 6$ $\qquad$ $8 \overset{\checkmark}{=} 8$

A graphical solution of these simultaneous equations is shown below. Observe it critically.

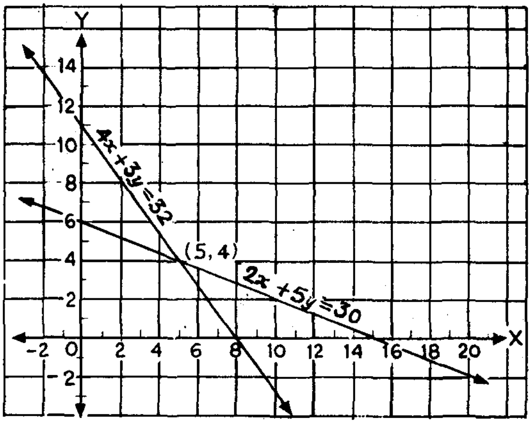


Fig. (6.1) (Graph)

Exercise 6.5

Solve the following equations :

(1) $3x + \frac{2y}{5} = 20$ $\quad ; \quad$ $\frac{x}{3} + y = 7$

(2) $\frac{5x}{3} - y = 3$; $\frac{x}{6} - \frac{y}{4} = 0$

(3) $\frac{x-3}{2} - \frac{y-2}{3} = 5$; $\frac{x-5}{5} - \frac{y+3}{4} = 0$

(4) $\frac{x+y}{2} = -5$; $\frac{3x+2y}{8} = -3$

(5) $\frac{x}{3} = \frac{y}{2}$; $\frac{2x}{15} = 4 - \frac{y}{5}$

(6) $\frac{x}{6} = y - 6$; $\frac{3x}{4} = 1 + y$

(7) $\frac{x-5}{3} = \frac{y+2}{7}$; $2x = 1 + 3y$

(8) $\frac{x}{3} + \frac{y}{4} = 2$; $3x + 4y = 25$

(9) $\frac{x}{2} + \frac{y}{3} = 2$; $2x + 3y = 13$

(10) $\frac{y}{2} - \frac{x}{3} = -1$; $\frac{y}{4} + \frac{x}{5} = 5$

§ 6.7 Simultaneous equations of the form $ax + by = p$; $bx + ay = q$

Observe the following simultaneous equations,

(1) $7x + 13y = 47$; $13x + 7y = 53$

(2) $32x + 33y = 31$; $33x + 32y = 34$

(3) $15x - 14y = 117$; $14x - 15y = 115$

In each example above it is observed that the coefficient of x in one equation is equal to the coefficient of y in the second equation and vice-versa. These equations can be solved by the usual method. But it is much easier to solve such equations by the method illustrated below.

In this method new simple equations are obtained by adding both the equations and by subtracting one equation from the other. Then a variable can be chosen so that it can be eliminated easily. Example given below will illustrate this point.

Example : $7x+13y = 47 \dots (\text{I})$ $\quad 13x+7y = 53 \dots (\text{II})$.

Let us add equations (I) and (II),

$$\begin{array}{rrcll} & 7x + 13y & = & 47 & \dots\dots\dots (\text{I}) \\ + & 13x + 7y & = & 53 & \dots\dots\dots (\text{II}) \\ \hline & 20x + 20y & = & 100 & \end{array}$$

$\therefore \quad x+y = 5 \qquad$ (dividing by 20)........(III)

Similarly let us subtract equation (II) from (I).

$$\begin{array}{rcll} 7x + 13y & = & 47 & \dots\dots\dots (\text{I}) \\ 13x + 7y & = & 53 & \dots\dots\dots (\text{II}) \\ - \quad - & & - & \\ \hline -6x + 6y & = & -6 & \end{array}$$

$\therefore \quad x-y = 1 \qquad$ (dividing by -6)........(IV)

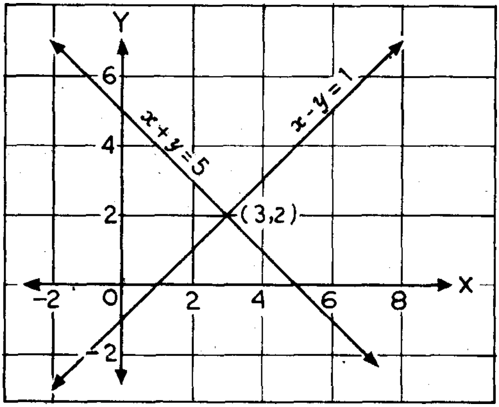


Fig. 6.2

Now let us add equations (III) and (IV)

$$\begin{array}{rcll} x+y & = & 5 & \dots\dots\dots(\text{III}) \\ x-y & = & 1 & \dots\dots\dots(\text{IV}) \\ \hline 2x & = & 6 & \end{array} \qquad \therefore \quad x = 3$$

Substituting $x = 3$ in equation (III) we get.

$3+y = 5 \quad \therefore \; y = 2$

Thus the solution set $\{(3, 2)\}$.

Check the answer yourself.

Exercise 6.6

Solve the following simultaneous equations,

(1) $32x+33y = 31$; $\quad 33x+32y = 34$

(2) $15x - 14y = 117$; $\quad 14x-15y = 115$

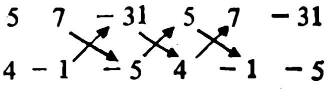
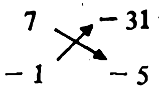
(3) $15x - 17y = 28$; $\quad 15y-17x+36 = 0$

(4) $10x - 40 = -15y$; $15x + 10y = 35$
(5) $14x + 41y = 96$; $41x + 14y = 69$
(6) $15x + 20y = 90$; $20x + 15y - 85 = 0$
(7) $21x + 22y = -3$; $22x + 21y = 3$
(8) $21x + 29y = 79$; $29x + 21y = 71$
(9) $29x + 37y = 53$; $29y + 37x = 13$
(10) $10x + 11y = 2$; $11x + 10y + 2 = 0$

§ 6.8 One more method of solving simultanous equations :

Example : Solve the following equations,

$5x + 7y = 31$........(I) $4x - y = 5$........(II)

Procedure	Working
Step (1) : Write the equations in the form $ax + by + c = 0$	(1) $5x + 7y - 31 = 0$ $4x - y - 5 = 0$
Step (2) : Write the coefficients of x, y and the constant term twice in order for each equation. This gives an arrangement of 2 rows and 6 columns.	(2) $\begin{array}{cccccc} 5 & 7 & -31 & 5 & 7 & -31 \\ 4 & -1 & -5 & 4 & -1 & -5 \end{array}$
Step (3) : Omit the first and last columns and draw the diagonal arrows as shown.	(3) 
Step (4) : Consider the elements of the first cross in the above arrangement. From the product of the elements at the ends of an arrow pointing downwards subtract the product of the elements at the ends of an arrow which points upwards. Call this number Dx.	(4)  $7 \times (-5) - (-1) \times (-31)$ $= -35 - 31$ $= -66$ $\therefore \quad Dx = -96$

Step (5) : Consider the elements of the second cross and repeat the working in step (4) above

Call this number Dy

Step (6) : Consider the elements of the third cross and repeat the working in step (4) above.

Call this number D.

Step (7) : Use the rule

$$\frac{x}{Dx} = \frac{y}{Dy} = \frac{1}{D},$$

to find the values of x and y, as

$$x = \frac{Dx}{D}, \quad y = \frac{Dy}{D}$$

(5)
$$\begin{matrix} -31 & & 5 \\ & \times & \\ -5 & & 4 \end{matrix}$$

$(-31) \times (4) - (-5) \times (5)$

$= -124 + 25$

$= -99$

$\therefore \quad Dy = -99$

(6)
$$\begin{matrix} 5 & & 7 \\ & \times & \\ 4 & & -1 \end{matrix}$$

$5 \times (-1) - (4) \times (7)$

$= -5 - 28$

$= -33$

(7) $\dfrac{x}{-66} = \dfrac{y}{-99} = \dfrac{1}{-33}$

$\therefore \quad x = \dfrac{-66}{-33}, \; y = \dfrac{-99}{-33}$

$\therefore \; x = 2 \quad , \quad y = 3$

Exercise 6.7

Solve the following simultaneous equations using the method illustrated above,

(1) $2x - y = -3$: $x - 2y = -3$

(2) $2x + y = 7$; $4x - y = 5$

(3) $4x + 7y = 2$; $3x + 5y = 1$

(4) $3x - 4y = 24$; $5y + 2x = -7$

(5) $3x = 4y - 6$; $5x - y = 7$

(6) $2x + y = -1$; $x - 4y = 13$

(7) $x + 2y = 6$; $4x + 3y = 4$

(8) $2x + y = 7$; $2x - y = 3$

Miscellaneous Exercise - 6 (A)

Solve the following equations :

(1) $3x - y = 11$; $x - y = 7$

(2) $4(x - y) = -20$; $\dfrac{5x}{8} + \dfrac{7y}{18} = 6$

(3) $\frac{x-2}{3} - \frac{3x-4}{4} = 1$

(4) $x - 2y = 14$; $y = x - 10$

(5) $9x - 7y = 15$; $7x - 9y = 1$

(6) $\frac{x-1}{2} - \frac{x-2}{3} = 1$

(7) $8x - 5y = 1$; $16(11x - 5y) = 11(3x + y)$

(8) $\frac{x+2y}{4} + \frac{2x-y}{2} = \frac{5}{2}$; $\frac{3x+y}{3} + \frac{x+y}{5} = 4$

(9) $\frac{2}{3}x + \frac{3}{2}y = 8\frac{1}{3}$; $\frac{3}{2}x + \frac{2}{3}y = 13\frac{1}{3}$

(10) $\frac{2(x-1)}{3} - \frac{3(x-2)}{4} = 1$

(11) $\frac{1}{2}x + \frac{1}{3}y = 4$, $\frac{3}{2}x - \frac{1}{3}y = 4$

(12) $y = 3x - 2$; $y = \frac{3x}{2} + 4$

(13) $\frac{p+1}{5} - \frac{3(p-1)}{10} = 2$

(14) $4(x - 2) = 5(1 - y)$; $3y + 2x = -24x - 4$

(15) $3(x + 3) = 2(x + y) + 4$; $3(2y - 3) - 3 = 4(2 + y - x)$

(16) $\frac{y}{2} - \frac{x}{3} = -1$; $\frac{y}{4} + \frac{x}{5} = 5$

(17) $\frac{2(p-1)}{5} - \frac{3(p+1)}{10} = p$

(18) $4x - 5y = 5$; $x + 7y = 26$

(19) $2x - 7y = 11$; $5x - 2y = 12$

(20) $\frac{2x-1}{3} - \frac{3x-1}{4} = 1$

(21) $12x + 13y = 51$; $13x + 12y = 49$

(22) $14x + 11y = 58$; $11x + 14y = 67$

(23) $\frac{x}{3} + \frac{y}{2} = 8$; $\frac{3x}{4} - \frac{y}{4} = 7$

(24) $\frac{3(x-2)}{2} - \frac{x-3}{4} = 2$

(25) $x + y = 0 \quad ; \quad \frac{x}{2} - \frac{y}{3} = -\frac{5}{3}$

(26) $x + \frac{y}{2} = 2 \quad ; \quad \frac{x}{2} - y + \frac{3}{2} = 0$

(27) $x + 7y = 23 \quad ; \quad 5x - 3y = 1$

(28) $\frac{p+2}{6} - \frac{p-3}{3} = p$

§ 6.9 To obtain equations :

In the nineth standard syllabus we have studied,

(1) Linear equtions in one variable

(2) Quadratic equations in one variable and

(3) Simultaneous equations in two variables.

We have also seen how equations play an important role in solving the problems in every day life.

Word problems contain some information and on the basis of this information we are asked to find answers to problems posed in them. The information given in word problems is of course numerical in nature. Since the information we are asked to find is not known at the outset, we use a letter or a variable to stand for this unknown. Since a variable stands for a number we use this fact to convert the information supplied in the problem into mathematical form. This mathematical form takes the form of an equation. In order to find the answer to the problem posed we have to solve this equation.

Addition, subtraction, multiplication and division are the four fundamental operations in mathematics. We call them binary operations because they can be performed only on two numbers at a time. Because of a binary operation, a number is associated with two given numbers. For example, $2 + 3 = 5$, $2 - 3 = -1$, $2 \times 3 = 6$, $2 \div 3 = \frac{2}{3}$.

In the above four examples, the pair of numbers 2 and 3 is associated with numbers 5, -1, 6 and $\frac{2}{3}$ respectively under the operations of addition, subtraction, multiplication and division.

But some operations can be performed on one number only. For example, doubling a number, taking three times a number, squaring a number, taking the cube of a number, taking the square - root of a number, etc. Since we are using variables for numbers, the above fundamental operations can be performed on two variables or on one variable and one number.

$x + 3$ means the sum of some number and 3. $2x$ means twice some number. In the same way $\frac{x}{7}$, $\sqrt{x}$, x^2 etc. can be interpreted.

While converting common problems which are given in words into mathematical form we are supposed to use numbers, operatical signs and signs of equality. If we are able to construct expressions which are stated to be equal in the given problem, it is possible for us to write the equation.

Form algebraic expressions for the following.

(1) Eight years more than Naseem's age.

Since we do not know Naseem's age, we assume that p stands for Naseem's age. Then $(p + 8)$ will express properly what is stated in words.

(2) Seven less than twice a number.

Here we do not know which number is meant. Hence we show the number by 'a' Therefore $(2a - 7)$ expresses in mathematical form what is stated in words.

(3) (i) 13 less than thrice a number.

(ii) 7 more than four times the same number.

(iii) the ratio of the above two expressions.

Above expressions take the following mathematical form.

(i) $3p - 13$ (ii) $4p + 7$ (iii) $\frac{3p - 13}{4p + 7}$

(4) The number in which the digit in the ten's place exceeds the digit in the unit's place by 2.

If p is taken to be the digit in the unit's place then the expression $10\ (p+2)+p$ will show the required number.

(5) Perimeter of a rectangle in which the length is twice the breadth.

If we assume r to the breadth then $2\ (r+2r)$, that is, $6r$ is the expression which denotes the perimeter of the rectangle.

(6) Area of a triangle in which the base is less than twice its height by 3.

Let the height be h. Then $\frac{1}{2}\ h\ (2h-3)$ is the expression which represents the area of the triangle.

(7) Sum of the angles of a triangle which are in the ratio 1 : 2 : 3

Let the measures of the angles of the triangle be x, $2x$ and $3x$. Then the sum of the angles of the triangle is $x+2x+3x$, that is $6x$. Thus $6x$ is the expression which denotes the sum of the angles of the required triangle.

(8) Average of the numbers in the ratio 3 : 4 : 5

If we assume $3x$, $4x$ and $5x$ as the numbers then their average is $\dfrac{3x+4x+5x}{3}$, that is $4x$. so $4x$ is the expression which denotes the average of the numbers in the ratio 3 : 4 : 5.

(9) The area of a trapezium in which the distance between the parallel sides is 5 cm and the greater of the parallel sides exceeds half the smaller by 8.

If the smaller parallel side is assumed to be x then area of the trapezium is $\frac{1}{2}\times 5\left(x+\dfrac{x}{2}+8\right)$, that is, $\dfrac{5\,(3x+16)}{4}$

(10) Sum of twice of a number and thrice of the another number.

Let x represent the first number and y represent the other. Then the required sum is $(2x+3y)$.

(11) In two rectangles the breadth of the first is twice the breadth of the other, length of the first is thrice the length of the other. Then the perimeter of the first rectangle.

Here if we assume l and b to be the length and the breadth respectively of the second rectangle, then the perimeter of the first rectangle is $2(2b+3l)$.

(12) The subtraction of five times the first number and four times the second number.

Let the first number be x and the second number be, y. Then the required number is $(5x-4y)$.

(13) Sum of the denominator and numerator of a fraction.

If we denote x for denominator and y for numerator then the expression showing their sum is $(x+y)$.

(14) The sum of the length of the base and the lengths of the congruent sides of an isosceles triangle.

Let x be the length of one of the congruent sides and y be the length of the base. Then the required expression for the sum of the sides is $(x+x+y)$, that is $(2x+y)$.

(15) Sum of Ashwini's age five years back and Manisha's age three years hence.

We assume Ashwini's present age as x and Manisha's present age as y. Then Ashwini's age 5 years back was $(x-5)$ years and Manisha's age 3 years hence will be $(y+3)$ years.

Hence the required sum of the ages is $[(x-5)+(y+3)]$ years, that is, $(x+y-2)$ years.

§ 6.10 Use of linear equations :

We are already familiar with the process of obtaining equations by converting into mathematical form, situations we meet in practical life.

Write down the equations of the following examples and solve them.

Exercise 6.8

(1) A number consists of two digits. The digit in the ten's place is twice the digit in the unit's place. The number obtained by interchanging the digits is less than the original number by 36. Find the original number.

(2) Son's present age is half the present age of his mother. Ten years back mother was thricc as old as her son. What are their present ages ?

(3) A boat takes thrice as much time to sail in the up-stream of a river as it would have taken to sail in the down-stream. If the speed of the water is 5 km / hour, find the speed of the boat in the still water.

(4) A number consists of two digits. The sum of the digits is 10. The number obtained by interchanging the digits exceeds the original number by 36. Find the original number.

(5) A number consists of two digits. The sum of its digits is 5. If 27 be added to the number, the digits get interchanged. Find the number.

(6) The digit in the ten's place of a two digit number is less than its digit in the unit's place by 4. The sum of its digits is one fourth of the number. Find the number.

(7) The sum of the digits of a two digit number is five. The digit obtained by increasing the digit in the ten's place by unity is one eighth of the number. Find the number.

(8) A number consists of two digits. The difference between its digits is 1. The number obtained by interchanging the digits is $\frac{5}{6}$ times the original number. Find the original number.

(9) The cost of a table is thrice the cost of a chair. A table and a chair together cost 500 rupees. What is the cost of the table ?

(10) Father is four times as old as his son. Ten years hence he will be two and half times as old as his son, what is the present age of the son ?

(11) The length of a rectangle is twice its breadth. If its perimeter is 90 cm find its length and breadth.

(12) The length of a rectangle is less than thrice its breadth by 5 decimetre If the perimeter of the rectangle is 70 decimetre, find its length and breadth.

(13) In $\triangle ABC$ the measure of $\angle B$ is twice the measure of $\angle C$. The measure of $\angle A$ is greater than the measure of $\angle B$ by 5°. Find the measure of each angle of $\triangle ABC$.

(14) In $\triangle PQR$, length of the side QR is less than twice the length of the side PQ by 2 cm Length of the side PR exceeds the length of the side PQ by 10 cm. If the perimeter of the triangle is 40 cm., find the length of each side of the triangle.

(15) The length of the rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increase 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length and breadth of the rectangle.

(16) Area of a square and a rectangle is same. The length and breadth of the rectangle are respectively 5 cm more and 4 cm. less than the side of the square. Find the side of the square.

§ 6.11 Problems on simultaneous equations :

Examples :

1. The sum of two numbers is 46. Greater number is greater than twice the smaller number by 1. Find the numbers.

Let the smaller number be x and the greater number be y.

Sum of the two numbers is 46

$\therefore \quad x + y = 46 \ldots\ldots\ldots\ldots (\mathrm{I})$

Greater number is greater than twice the smaller number by 1,

$\therefore \quad y = 2x + 1 \ldots\ldots\ldots\ldots (\mathrm{II})$

Substituting $2x + 1$ for y in equation (I) we have,

$x + 2x + 1 = 46$

$\therefore \quad 3x + 1 = 46$

$\therefore \quad 3x = 45 \qquad \therefore \quad x = 15$

Now substituting $x = 15$ in equation (II),

$y = 2 \times 15 + 1 \qquad \therefore \quad y = 31$

$\therefore$ Smaller number is 15 and greater number is 31.

2. The sum of the present ages of Suresh and his father is 89 years. After 11 years father's age will be twice Suresh's age. Find their present ages.

Let the present age of Suresh be x years and that of his father be y years.

Sum of their present ages is 89,

$\therefore \quad x+y = 89 \ldots\ldots\ldots\ldots (\text{I})$

Eleven years hence Suresh's age $= (x+11)$ years and his father's age $(y+11)$ years,

But 11 years hence father is twice as old as his son.

$\therefore \quad y+11 = 2(x+11)$

$\therefore \quad y+11 = 2x+22$

$\therefore \quad y = 2x+11 \ldots\ldots\ldots\ldots (\text{II})$

Now we will consider the two equations together and find their solution.

Substituting $(2x+11)$ for y in equation (I) we have,

$x+2x+11 = 89$

$\therefore \quad 3x = 78$ (Subtracting 11 from both sides)

$\therefore \quad x = 26$

Substituting the value of x in equation (I), we have,

$26+y = 89 \qquad \therefore \quad y = 63$

$\therefore$ Suresh's present age is 26 years and that of his father's is 63 years.

Exercise 6.9

(1) The sum of the two numbers, is 45. The greater number exceeds thrice the smaller number by 5. Find the numbers.

(2) The sum of the two numbers is 22. The greater number is less than twice the smaller number by 2. Find the numbers.

(3) The sum of the two numbers is 95. Three times the smaller number equals twice the greater number. Find the numbers.

(4) The sum of two numbers is 125. Their difference is 25. Find the numbers.

(5) The difference between two numbers is 40 and three times the smaller is eqaal to twice the larger. Find the numbers.

(6) A string 12 metres long is cut in to two pieces such that one part is greater than the other by two metres. Find the length of each part.

(7) Sum of the present ages of Ramesh and his father is 43 years. Six years hence father will be four times as old as his son then. Find their present ages.

(8) Present age of a person is four times his son. Eighteen years hence his age will be twice the age of his son then. Find their present ages.

(9) Sum of the present ages of Umesh and his mother is 60 years. Five years back his mother's age was four times his age at that time. Find their present ages.

3. Twelve chairs and six tables together cost Rs. 2388. If instead six chairs and twelve tables were purchased, the cost together would have been Rs. 3480. Find the cost of each chair and each table.

Let the cost of a chair be x rupees and that of a table be y rupees.

Using first condition we have,

$12x + 6y = 2388 \dots\dots\dots\dots (I)$

Using second condition we have.

$6x + 12y = 3480 \dots\dots\dots\dots (II)$

Adding equations (I) and (II) we have,

$18x + 18y = 5868$

$\therefore$ dividing by 18 we have,

$x + y = 326 \dots\dots\dots\dots (III)$

Again subtracting (II) from (I), we have,

$6x - 6y = -1092$

$\therefore$ dividing by 6 we have,

$x - y = -182 \dots\dots\dots\dots (IV)$

Adding (III) and (IV), we have,

$2x = 144 \quad \therefore \quad x = 72$

Substituting this value of x in (III), we have,

$72 + y = 326 \quad \therefore \quad y = 254$

$\therefore$ Each chair costs Rs. 72 and each table costs Rs. 254.

4. Out of 765 km journey a part was made by motorcar at a

speed of 60 km per hour. The remaining journey was made by train at a speed of 96 km per hour. If the total distance was covered in 9 hours, find the distance travelled by the train and the time taken by it to cover that distance.

Let the time taken by the car be x hrs. and that by train be y hrs. Then, from the second condition we have,

$x + y = 9$.................(I)

and from the first condition we have,

$60x + 96y = 765$............(II)

Multiplying equation (I) by 60 and subtracting it from (II) we have,

$$\begin{array}{r} 60x + 96y = 765 \\ \underset{-}{60x} + \underset{-}{60y} = \underset{-}{540} \\ \hline 36y = 225 \end{array}$$

$$\therefore \quad y = \frac{225}{36} = \frac{25}{4} = 6\frac{1}{4}$$

and $96\,y = 96 \times \frac{25}{4} = 600$

$\therefore$ the distance covered by the train is 600 km in 6 hours and 15 minutes.

Exercise 6.10

(1) 15 oranges and 8 apples together cost Rs. 19·25. If instead, 8 oranges and 15 apples were purchased, they together would have cost Rs. 21/– Find the price of each orange and each apple.

(2) A school requested 150 copies of Algebra books and 140 copies of Geometry books from the Text-book Bureau,together costing Rs. 1600/–. After a week the school further requested 120 copies of Algebra books and 130 copies of Geometry books. This time they had to pay Rs. 1370. What is the price of Algebra book and Geometry book per copy each ?

(3) If 1 is added to the numerator of a fraction, the resulting fraction has value $\frac{3}{4}$. On the other hand if 2 be added to the

denominator of that fraction, the resulting fraction is of the value $\frac{1}{2}$. Find the original fraction.

(4) It took 8 hours to travel a distance from one city to the other by car. If the speed of the car was increased by 8 km per hour, the same distance would have been covered in one hour less. Find the distance between the two cities.

(5) On the opening day of the sale of tickets of a particular drama, in all 105 tickets of Rs. 10 and Rs. 8 were sold out. If the amount collected on that day was Rs. 922, find the number of tickets sold out of each category.

(6) You know that the sum of the measures of two consecutive angles of a parallelogram is 180°. In a certain parallelogram measure of one of the consecutive angles exceeds the other by 20°. Find the measure of each angle of the perallelogram by forming simultaneous equations.

5. The perimeter of a rectangle is 28 cm. Its length exceeds twice its breadth by 5 cm. Find the length and the breadth of the rectangle.

Let the length and the breadth of the rectangle be x cm. and y cm **respectively.**

$\therefore$ perimeter of the rectangle is

$$x + y + x + y = 2x + 2y$$
$$= 2(x + y) \text{ cm.}$$

But the given perimeter is 28 cm.

$$\therefore\ 2(x + y) = 28$$
$$\therefore\ x + y = 14 \ \ldots\ldots\ldots\ldots \ (\text{I})$$

The second condition gives

$$x = 2y + 5 \ \ldots\ldots\ldots\ldots\ldots\ldots \ (\text{II})$$

Substituting the value of x from equation (II) in (I) we have,

$$2y + 5 + y = 14$$
$$\therefore\ 3y = 14 - 5$$
$$\therefore\ 3y = 9 \qquad \therefore\ y = 3$$

Now, substituting $y = 3$ is equation (II) we have,

$$x = 2 \times 3 + 5 \qquad \therefore\ x = 11$$

$\therefore$ the length of the rectangle is 11 cm. and its breadth is 3 cm.

Exercise 6.11

In the following examples form simultaneous equations and solve them.

(1) In $\triangle ABC$, $m\angle B = 90^\circ$. $m\angle A$ exceeds $m\angle C$ by 10°. Find the measure of each angle of $\triangle ABC$.

(2) The perimeter of a rectangle is 180 cm. Its length is greater than twice its breadth by 9 cm. Find the length and breadth of the rectangle.

(3) In a right triangle sum of the lengths of its perpendicular sides is 84 cm. If the difference between these sides is 12 cm find the length of each side of the right triangle.

(4) In a cyclic quadrilateral the difference between the measures of a pair of the opposite angles is 12°. What is the measure of each angle in that pair ?

Miscellaneous Exercise – 6 (B)

1. If the numerator of a fraction is reduced by 1, its value becomes $\frac{1}{2}$ and if the denominator is increased by 7, its value becomes $\frac{1}{3}$. Find the fraction.
2. A two–digit number is six times the sum of its digits. If the digit in the units' place is increased by 2 and the digit in the ten's place is decreased by 2, then the number so formed is 4 times the sum of its digits. Find the original number.
3. A sum of money was distributed equally among a certain number of children. If there were 10 children more, each one would have got a rupee less. But if there were 15 children less, each one would have got 3 rupees more. Find the sum of money distributed and also the number of children.
4. A person bought a horse and a buggie together for Rs.3500 After some months. he sold the horse at a gain of 25% and the buggie at a gain of $16\frac{1}{4}\%$. Thus he gained 20% on the whole. Find the cost price of the horse.

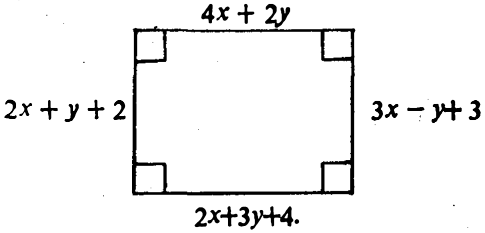
5. Children were fallen in for drill. If each row contained 4 children less, 10 more rows would have been made. But if 5 more children were fallen in each row, the number of rows would have reduced by 8. How many children were there ?

6. Last year, one kilogram of tea and 3 kilograms of sugar together cost Rs. 21. This year the rate of tea increased by 10% and that of sugar by 20%. So, the same amounts of tea and sugar now cost Rs. 23.45. Find the present rates of tea and sugar per kilogram.

7. If the average speed of an aeroplane is reduced by 60 kilometres per hour, it would take 24 minutes more to go a certain distance. But if the speed is inereased by 40 kilometres per hour, it would take 12 minutes less to go the same distance. Find the average speed of the aeroplane in kilometres per hour.

8. If the length of a rectangle is reduced by 1 cm. and breadth increased by 1 cm, its area would increase by 2 sq. cm. But if the length is incercsed by 2 cm, and breadth is reduced by 1 cm, the area would be increased by 7 sq. cm. Find the length and the breadth of the rectangle.

9. In the adjoining figure the measures of the sides of a rectangle are shown. Find the length and the breadth.



10. Five tables and 4 chairs together cost Rs. 1700. But, if the numbers of tables and chairs were interchaged, the cost would have reduced by Rs. 115. Find the cost of each chair.

11. The sum of the ages of two brothers is 43 years. Eleven years hence the age of the elder brother would be $\frac{7}{6}$ times the then age of the younger. Find their present ages.

12. Ashok scored first rank in some items and came up second in some items of the annual sports. The first rank scored

5 points while the second place 3 points. Ashok scored 34 points. If the number of items in which he stood first had been the number of items of his ranking second and the number of items in which he got second place was the number of his ranking first, his score would have reduced by 4 points. Find the number of items in which he stood first.

13. The difference between two numbers is 8 and ten times the smaller is equal to eight times the greater. Find the numbers.
14. Post cards at 15 paise each and inland letters at 35 paise each were purchased. In all they were 44 and cost Rs. 11.40. But if the numbers of the cards and letters were interchanged how much less amount would have been spent ?
15. A passenger is allowed some free luggage in a railway journey. Two passengers had together 80 kilograms of luggage and they were charged Rs. 6 and Rs. 10 respetively for the excess luggage. If the whole luggage had belonged to one passenger he would have to pay Rs. 28 as excess luggage charge. Find how much luggage is allowed free.
16. The perimeter of an isosceles triangle is 18 cm. If the length of each of the congruent sides had been the length of the base and the length of the base had been the length of each of the two congruent sides, the perimeter would have inereased by 3 cm. Find the lengths of the sides of the triangle.

● ● ● ●

Chapter 7

Quadratic Equations

§ 7.1 Revision :

You studied ' quadratic equations ' in the ninth standard. The general form of a quadratic equation is $ax^2 + bx + c = 0$, $(a \neq 0)$; a, b, c being real numbers. It is obvious that if $a = 0$, the equation will cease to be a quadratic equation.

After substituting a value of x in the quadratic expression ax^2+bx+c, if the quadratic expression reduces to zero, the equation is said to be satisfied by that value. The set of such values of 'x' which satisfy the equation is called the 'solution set' or the 'truth set' of that equation. To solve a quadratic equation means to find out its solution set.

It is quite easy to solve the equation $ax^2+bx+c=0$, if it is possible to factorise the given quadratic expression ax^2+bx+c.

Read the following example.

$$2x^2+x-3=0$$

In this example, the constant term, -3, is a negative number. So let us see if the difference between the factors of 3×2 is 1. $3-2=1$. Therefore, the expression can be factorised.

$$\begin{aligned} 2x^2+x-3 &= 2x^2+3x-2x-3 \\ &= x(2x+3)-1(2x+3) \\ &= (2x+3)(x-1) \end{aligned}$$

$\because \quad (2x+3)(x-1)=0$

$\therefore \quad 2x+3=0 \quad$ or $\quad x-1=0$

$\therefore \quad x=-\frac{3}{2} \quad$ or $\quad x=1$

$\therefore$ the solution set is $\left\{-\frac{3}{2}, 1\right\}$

Check :

(1) Substituting $x=1$ in the given equation.

$$2(1)^2+1-3 \overset{?}{=} 0$$

$$\therefore \quad 2+1-3 \overset{?}{=} 0$$

$$\therefore \quad 0 \overset{\checkmark}{=} 0$$

(2) Substituting $x=-\frac{3}{2}$ in the given equation,

$$2\left(-\frac{3}{2}\right)^2+\left(-\frac{3}{2}\right)-3 \overset{?}{=} 0$$

$$\therefore \quad 2\times\frac{9}{4}-\frac{3}{2}-3 \overset{?}{=} 0$$

$$\therefore \quad \frac{9}{2}-\frac{3}{2}-\frac{3}{1} \overset{?}{=} 0$$

$$\therefore \quad \frac{9-3-6}{2} \overset{?}{=} 0$$

$$\therefore \quad 0 \overset{\checkmark}{=} 0$$

Exercise 7.1

Find the solution sets of the following quadratic equations using the method of factorization. Check your answers by substituting the values of the variables in the respective equations,

(1) $3x^2 + 7x + 2 = 0$

(2) $2x^2 - 9x + 10 = 0$

(3) $2y^2 + 15y + 18 = 0$

(4) $3m^2 - 10m + 8 = 0$

(5) $4x^2 - 11x + 6 = 0$

(6) $4p^2 + 11p + 6 = 0$

(7) $8x^2 + 10x - 3 = 0$

(8) $5x^2 - 11x + 6 = 0$

(9) $6y^2 + 17y + 12 = 0$

(10) $6x^2 - 19x + 10 = 0$

(11) $6x^2 - 5x - 6 = 0$

(12) $2y^2 - 5y - 7 = 0$

(13) $6x^2 + 11x - 10 = 0$

(14) $2y^2 - y - 3 = 0$

(15) $2m^2 + 11m + 15 = 0$

(16) $7x^2 - 19x - 6 = 0$

(17) $12y^2 + 11y + 2 = 0$

(18) $6x^2 - x - 12 = 0$

(19) $5x^2 + 17x + 6 = 0$

(20) $3y^2 - 7y - 6 = 0$

§ 7.2 The equation $ax^2 + bx = 0$

If $c = 0$, the quadratic equation $ax^2 + bx + c = 0$ reduces to the form $ax^2 + bx = 0$. The expression $ax^2 + bx$ can be easily factorized.

Example :– Solve : $2x^2 - 18x = 0$

$$2x^2 - 18x = 0$$

$$\therefore \quad 2x(x - 9) = 0$$

$$\therefore \quad 2x = 0 \quad \text{or} \quad x - 9 = 0$$

$$\therefore \quad x = 0 \quad \text{or} \quad x = 9$$

$\therefore$ $\{0, 9\}$ is the solution set.

Check :

(1) Substituting $x = 9$ in the given equation,

$$2(9)^2 - 18(9) \overset{?}{=} 0$$

$$\therefore \quad 2 \times 81 - 162 \overset{?}{=} 0$$

$$\therefore \quad 162 - 162 \overset{?}{=} 0$$

$$\therefore \quad 0 \overset{\checkmark}{=} 0$$

(2) Substituting $x = 0$ in the given equation,

$$2(0)^2 - 18(0) \overset{?}{=} 0$$

$$\therefore \quad 2 \times 0 - 18 \times 0 \overset{?}{=} 0$$

$$\therefore \quad 0 - 0 \overset{?}{=} 0$$

$$\therefore \quad 0 \overset{\checkmark}{=} 0$$

Exercise 7.2

Find out the solution sets of the following quadratic equations by using the method of factorization.

(1) $x^2 = 6x$ (2) $3x^2 - 5x = 0$ (3) $4y^2 + 7y = 0$

(4) $9x^2 = 5x$ (5) $2x^2 = 5x$ (6) $9m^2 - 12m = 0$

(7) $4x^2 - 4x = 0$ (8) $4x^2 + 16x = 0$ (9) $2x^2 - 6x = 0$

(10) $3x^2 - 3x = 0$ (11) $8y^2 = -6y$ (12) $6p^2 = -5p$

(13) $2x^2 - 3x = 0$ (14) $2x^2 + 10x = 0$ (15) $4y^2 - 20y = 0$

§ 7.3 The equation $ax^2 + c = 0$

If $b = 0$, the equation $ax^2 + bx + c = 0$ reduces to the form $ax^2 + c = 0$. If 'c' is negative and a is positive, subtracting c from both sides we get,

$$ax^2 = -c$$

Then dividing both sides by a we get

$$x^2 = -\frac{c}{a}$$

Now if we take square root of both sides we get the solution set.

Study the following example,

$$3x^2 - 243 = 0$$

$\therefore$ $3x^2 = 243$ (adding 243 on both sides)

$\therefore$ $x^2 = 81$ (dividing both sides by 3)

$\therefore$ $x = 9$ or $x = -9$

So, $\{ 9, -9 \}$ is the solution set.

Check :

(1) Substituting $x = 9$ in the given equation,

$$3(9)^2 - 243 \overset{?}{=} 0$$

$$3 \times 81 - 243 \overset{?}{=} 0$$

$$243 - 243 \overset{?}{=} 0$$

$$0 \overset{\checkmark}{=} 0$$

(2) Substituting $x = -9$ in the given equation.

$$3(-9)^2 - 243 \overset{?}{=} 0$$

$$3 \times 81 - 243 \overset{?}{=} 0$$

$$243 - 243 \overset{?}{=} 0$$

$$0 \overset{\checkmark}{=} 0$$

Exercise 7.3

Find out the soluion sets of the following quadratic equations,

(1) $4x^2 = 16$ (2) $\frac{y^2}{5} = 5$ (3) $9x^2 - 1 = 0$

(4) $x^2 - 3 = 0$ (5) $(m+1)^2 - 36 = 0$ (6) $(x+3)^2 - 7 = 0$

(7) $(n+5)^2 - 64 = 0$ (8) $(3y-1)^2 - 18 = 0$ (9) $(x+\frac{1}{4})^2 = 1$

(10) $(p-2)^2 - 12 = 0$ (11) $9x^2 - 16 = 0$ (12) $x^2 - 4a^2 = 0$

(13) $(y-2)^2 = 16$ (14) $3(x+3)^2 = 75$ (15) $(2m+3)^2 = 81$

(16) $\frac{1}{2}(x+1)^2 = 18$ (17) $\frac{1}{4}(x+5)^2 = 9$ (18) $\frac{1}{3}(y-1)^2 = 27$

(19) $2(x-7)^2 = 8$ (20) $\frac{1}{5}(2x-1)^2 = 20$

§ 7.4 Another method of solving the given quadratic equation :

Sometimes it is not possible to factorize the given quadratic expression $ax^2 + bx + c$ easily. In such cases we add or subtract a suitable number in the given quadratic expression. Then we rearrange its terms in such a way that some terms form a complete square. This immediately helps to find the solution of the given equation.

Example :

1. Fill in the blanks in the following example,

$x^2 + 8x - 9 = 0$

$\therefore (x + \square)^2 - 9 - \square = 0$

$\therefore (x + \square)^2 = \square + 9$

$\therefore x + \square = \pm\sqrt{\square + 9}$

$\therefore x = \pm$ ______________

2, Study the following example,

$2x^2 + 5x + 1 = 0$

It is not easy to factorize the expression $2x^2 + 5x + 1$. Therefore, solve this equation as shown below,

$$2x^2 + 5x + 1 = 2\left(x^2 + \frac{5x}{2} + \frac{1}{2}\right)$$

$$= 2\left[\left(x+\frac{5}{4}\right)^2 + \frac{1}{2} - \frac{25}{16}\right]$$

$$= 2\left[\left(x+\frac{5}{4}\right)^2 - \frac{17}{16}\right]$$

$$\therefore\ 2\left[\left(x+\frac{5}{4}\right)^2 - \frac{17}{16}\right] = 0$$

$$\therefore\ \left(x+\frac{5}{4}\right)^2 - \frac{17}{16} = 0 \qquad [\text{dividing by } 2]$$

$$\therefore\ \left(x+\frac{5}{4}\right)^2 = \frac{17}{16}$$

$$\therefore\ x+\frac{5}{4} = \pm\sqrt{\frac{17}{16}}$$

$$\therefore\ x+\frac{5}{4} = \pm\frac{\sqrt{17}}{4}$$

$$\therefore\ x = -\frac{5}{4}+\frac{\sqrt{17}}{4} \quad \text{or} \quad x = -\frac{5}{4}-\frac{\sqrt{17}}{4}$$

$$\therefore\ x = \frac{-5+\sqrt{17}}{4} \quad \text{or} \quad x = \frac{-5-\sqrt{17}}{4}$$

$$\therefore\ \left\{\frac{-5+\sqrt{17}}{4}, \frac{-5-\sqrt{17}}{4}\right\} \text{ is the solution set.}$$

3. Fill in the following gaps appropriately,

$$ax^2 + bx + c = 0$$

$$\therefore\ a\left(x^2 + \frac{b}{\square}x + \frac{c}{\square}\right) = 0$$

$$\therefore\ \left[x^2 + \frac{b}{\square}x + \frac{c}{\square}\right] = 0 \qquad [\because\ a \neq 0]$$

$$\therefore\ \left(x+\frac{b}{\square}\right)^2 + \frac{c}{\square} - \frac{b^2}{\square} = 0$$

$$\therefore\ \left(x+\frac{b}{\square}\right)^2 = \frac{b^2}{\square} - \frac{c}{\square}$$

$$\therefore\ \left(x+\frac{b}{\square}\right)^2 = \frac{b^2-4ac}{\square}$$

$$\therefore \; x + \frac{b}{\square} = \frac{\pm\sqrt{b^2 - 4ac}}{\square}$$

$$\therefore \; x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Solve : $2x^2 - x - 4 = 0$

In this example, $a = 2, b = -1, c = -4$

$\therefore$ as per formula obtained in ex. 3 above,

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2\ (2)}$$

$$= \frac{1 \pm \sqrt{1 + 32}}{4}$$

$$\therefore \; x = \frac{1 + \sqrt{33}}{4} \qquad \text{or } x = \frac{1 - \sqrt{33}}{4}.$$

[**Note** : Consider the equation $2x^2 + x + 3 = 0$. Here, $a = 2$, $b = 1$ and $c = 3$.

$\therefore$ as per formula above, $x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(2)(3)}}{2\ (2)}$

$$= \frac{-1 \pm \sqrt{-23}}{4}$$

These are the solutions of the given equation. But $\sqrt{-23}$ is not a real number.So these solutions are not real numbers.

This means, that not all quadratic equations have solutions in the set of real number.]

Exercise 7.4

Find the solution sets of the following eqations.

(1) $x^2 + 6x - 2 = 0$

(2) $x^2 - 4x + 1 = 0$

(3) $3x^2 + 7x + 3 = 0$

(4) $5x^2 = 2x + 2$

(5) $6x^2 - 7x - 1 = 0$

[6] $x^2 + 3x - 1 = 0$

§ 7.5 Problems based on quadratic equations

Examples :

1. Sum of the squares of two consecutive natural numbers is 61. Find the numbers.

Let the first natural number be x.

$\therefore$ the next consecutive natural number will be $(x+1)$

The sum of the squares of these numbers is 61. Hence the required equation is,

$$x^2+(x+1)^2=61$$

$\therefore$ $x^2+x^2+2x+1=61$

$\therefore$ $2x^2+2x-60=0$

$\therefore$ dividing both the sides by 2

$x^2+x-30=0$.

$\therefore$ $(x+6)(x-5)=0$

$\therefore$ $x+6=0$ or $x-5=0$

$\therefore$ $x=-6$ or $x=5$

$\therefore$ the first number is -6 or 5.

Since -6 is not *a* natural number. the required consecutive numbers will be 5 and 6.

Check :

Sum of the squares of the numbers is

$$(5)^2+(6)^2=25+36=61$$

Exercise 7.5

(1) Sum of the squares of three natural consecutive numbers 29. Find the middle number.

(2) Sum of the square of a positive real number and thrice that number is 70. Find the number.

(3) Sum of a non-zero number and its reciprocal is $\frac{13}{6}$. Find the number.

(4) Difference between a number and its reciprocal is $\frac{80}{9}$. Find the number.

(5) Product of two consecutive even numbers is 48. Find the numbers.

(6) Product of two consecutive odd numbers is 143. Find the numbers.

(7) Two natural numbers differ by 3. Their product is 180. Find the numbers.

(8) Sum of the ages of father and his son is 45. Product of their ages is 200. Find their present ages.

(9) The digit in the ten's place of a two digit number is equal to the square of the digit in the unit's place. If 54 be subtracted from that number its digits are interchanged. Find that number.

(10) Three consecutive odd numbers are such that the sum of the squares of the first two numbers is greater than the square of the third by 65. Find the numbers.

2. The length of a rectangular plot is greater than thrice its breadth by 2 metres. The area of the plot is 120 sq. metres. Find the length and breadth of the plot.

Let us assume the breadth of the rectangle to be x metres.

$\therefore$ its length $= (3x + 2)$ mtrs.

Area of the rectangle = length × breadth

$= (3x + 2) \times x$

$\therefore$ the required equation is

$(3x + 2) \times x = 120$

i. e. $3x^2 + 2x = 120$

i e. $3x^2 + 2x - 120 = 0$

$\therefore (3x + 20)(x - 6) = 0$

$\therefore 3x + 20 = 0$ or $x - 6 = 0$

$\therefore x = -\frac{20}{3}$ or $x = 6$

$\therefore$ the breadth of the rectangle $= -\frac{20}{3}$ or 6 metres.

But breadth cannot be a negative number.

$\therefore$ the breadth of the rectangle = 6 metres.

$\therefore$ length $= 3 \times 6 + 2 = 20$ metres.

Check :

Area = length × breadth

$= 20 \times 6 \quad = 120$ sq. metres.

Exercise 7.6

(1) The length of a rectangular ground is greater than twice its breadth by 5 metres. Area of the ground is 1125 sq. metres. Find the length and the breadth of the ground.

(2) The length of a rectangular plot is thrice its breadth. Its area is 768 sq. metres. Find its length.

(3) Perimeter of a rectangle is 44 cm. Its area is 120 sq. cms. Find the length and breadth of the rectangle.

(4) Area of a rectangular hall is 48 sq. mtrs. If the length and breadth be increased by 1 mtr. each, the area is increased by 15 sq. mtrs. Find the length and the breadth of the hall.

(5) Perimeter of a rectangular playground is 60 metres. Its area is 216 sq. mtrs. Find its length and breadth.

(6) The length of a rectangle is greater than its breadth by 5 mtrs. Its area is 66 sq. mtrs. Find its length.

(7) Difference between the length and breadth of a rectangular garden is 2 metres. Its area is 120 sq. mtrs. Find its length and breadth.

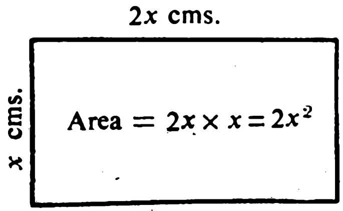
(8) One side of a right triangle (excluding the hypotenuse) is greater than the other by 2 cm. Area of the triangle is 24 sq. cm. Find the lengths of the perpendicular sides. (Area of a right triangle $= \frac{1}{2} \times$ product of the lengths of perpendicular sides.)

(9) Sum of the lengths of the perpendicular sides of a right triangle is 42 cms. Length of the hypotenuse is 30 cms. Find the lengths of the perpendicular sides.

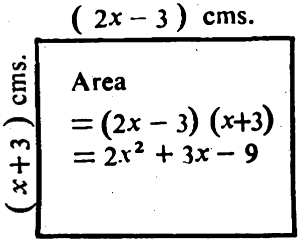
[Use : $(\text{hypotenuse})^2 = (\text{base}) + (\text{height})^2$]

(10) One perpendicular side of a right triangle is greater than thrice the other perpendicular side by 3 cms. The length of the hypotenuse is 25 cms. Find the length of the perpendicular sides.

3. Length of a rectangle is twice its breadth. If the length is reduced by 3 cms. and breadth be increased by 3 cms. the area increases by 27 sq. cms. Find the length and the breadth of the rectangle.



(Fig. 1)



(Fig. 2)

Assume the breadth of the rectangle in fig. 1 to be x. Therefore its length $= 2x$

$\therefore$ Area $= 2x \times x = 2x^2$ sq. cms.

Breadth of the rectangle in fig. $2 = (x + 3)$ cms. and length $= (2x - 3)$ cms.

Relation between the areas of the two rectangles is given

$\therefore$ the required equation is

area of the rectangle (fig. 1) + 27

= area of the rectangle (fig. 2)

$\therefore 2x^2 + 27 = (2x - 3)(x + 3)$

$\therefore 2x^2 + 27 = 2x^2 + 3x - 9$

Adding $(-2x^2 + 9)$ to both sides.

$36 = 3x \qquad \therefore x = 12$

$\therefore$ the breadth of the rectangle = 12 cm. and the length $= 2 \times 12 = 24$ cms.

Chcek

Area of the first rectangle $= 12 \times 24 = 288$ sq. cm.

Length of second rectangle $= 2 \times 12 - 3 = 21$ cms.

Breadth of second rectangle $= 12 + 3 = 15$ cms.

Area of second rectangle $= 21 \times 15 = 315$ sq. cm.

$288 + 27 = 315$

Exercise 7.7

(1) Length of a rectangle is thrice its breadth. If the length is reduced by 3 cms. and the breadth is increased by 3 cms., the area increases by 21 sq. cms. Find the length and the breadth of the rectangle.

(2) Base of a triangle is $2\frac{1}{2}$ times its altitude. Keeping the length of the base constant if the length of the altitude be increased by 2 cms. The area increases by 15 sq. cms. Find the base and altitude of the triangle.

(3) Area of a square is greater than the area of another square by 24 sq. cms. If the difference between their sides is 2 cms., find the sides of the two squares.

(4) Length of one diagonal of a rhombus is greater than that of the other diagonal by 5 cms. The area of the rhombus reduces by 24 sq. cms. if the lengths of the diagonals are deduced by 3 cms. each. Find the lengths of the diagonals.

Miscellaneous Exercise - 7

(1) The sum of two natural numbers is 15 and the sum of their squares is 117. Find the numbers.

(2) The difference between the lengths of the two sides of a right triangle containing the right angle is 7 and the area of the triangle is 60. Find the length of the hypotenuse.

(3) The sum of the squares of three consecutive natural numbers is 365. Find the middle number.

(4) The area of a rectangular garden is 432 sq. metres and the length of the fencing around it is 86 metres. Find the length and the breadth of the garden.

(5) If from the square of a number is subtracted nine times its next consecutive number, the remainder is 81. Find the number.

(6) The sum of the digits in a two-digit number is 9 and the product of the digits is 20. How many such numbers are there and what are they ?

(7) 500 bottles of a drug were packed equally in some containers. The number of bottles contained in each container was less by 5 than the number of containers required. How many bottles were packed in each container ?

(8) The distance between two opposite corners of a rectangular field is 34 metres and the length of the field exceeds its breadth by 14 metres. Find the area of the field.

(9) When the children in a school were arranged for a mass drill in equal rows, there were 5 rows more than the children in each row. If the roll call that day was 750, how many rows were formed ?

(10) The perimeter of a right triangle is 56 cms. and the difference between the lengths of sides containing the right angle is 17 cms. Find the length of the hypotenuse of the triangle.

(11) The sum of the squares of two consecutive odd numbers is greater by 2 than 16 times their sum. Find the numbers.

(12) The number denoting the area of a square is the same as the number denoting its perimeter. What is the area of the square ?

(13) In a mango-grove there are 4 more trees in each vertical row than in every horizontal row. If the total number of trees is 480, find the number of trees in each horizontal row.

(14) A fraction and its reciprocal add up to $\frac{25}{12}$. Find the fraction. How many such fractions are there ?

(15) The product of four consecutive positive numbers is 840. Find the largest of the numbers.

● ● ● ●

Chapter 8

Polynomials

§ 8.1 Revision

You have learnt about polynomials. $3x$, $-4x^2y$, $\frac{1}{5}abc$, -5, 0, $3x+y$, $x^2-5x+24$, $a^2+2ab+b^2$, a^3-5a^2+7 are all polynomials.

$3x^{-2}$ is not a polynomial because the index of the variable x is a negative integer, Similarly $a^{\frac{2}{3}}+a^{2}$ is not a polynomial because the index of a in the first term is a fraction. Indices of variables in a polynomial are always non-negative integers.

Addition and Subtraction of Polynomials :

1. Add $5a^2b^2-7ab^3+5ab$ and $6ab^3-5ab^2-4a^2b^2$ while adding or subtracting polynomials, like terms should be brought to-gether.

$(5a^2b^2-7ab^3+5ab)+(6a^3-5ab^2-4a^2b^2)$

$=(5a^2b^2-4a^2b^2)+(-7ab^3+6ab^3)+5ab-5ab^2$

...... collecting like terms.

$=a^2b^2-ab^3+5ab-5ab^2$

We now add the above polynomials columnwise.

$$\begin{array}{rrrrr} & 5a^2b^2 & -7ab^3 & +5ab & \\ + & -4a^2b^2 & +6ab^3 & & -5ab^2 \\ \hline & a^2b^2 & -ab^3 & +5ab & -5ab^2 \end{array}$$

2. Subtract $3x^3y+5xy^3-7xy+8$ from $6x^3y-5x^2y^2-2xy^3$,

$(6x^3y-5x^2y^2-2xy^3)-(3x^3y+5xy^3-7xy+8)$

$=6x^3y-5x^2y^2-2xy^3-3x^3y-5xy^3+7xy-8$

$=\underline{6x^3y-3x^3y}-5x^2y^2\underline{-2xy^3-5xy^3}+7xy-8$

...... collecting like terms

$=3x^3y-5x^2y^2-7xy^3+7xy-8$

Note that the sign of each term in the second polynomial is reversed.

Subtracting columnwise

$$\begin{array}{rrrrr} 6x^3y & -\ 5x^2y^2 & -\ 2xy^3 & & \\ \underline{\overset{}{\underset{-}{}}\ 3x^3y} & \underline{\qquad\qquad} & \underline{\overset{+}{\underset{-}{}}\ 5xy^3} & \underline{\overset{-}{\underset{+}{}}\ 7xy} & \underline{\overset{+}{\underset{-}{}}\ 8} \\ 3x^3y & -\ 5x^2y^2 & -\ 7xy^3 & +\ 7xy & -\ 8 \end{array}$$

3. If $A = (a^2 + 3ab + b^2)$ and $B = (2a^2 - 2ab + 4b^2)$ then express (I) $A + B$ and (II) $A - B$ in the simplest form.

$$\therefore\ A + B = (a^2 + 3ab + b^2) + (2a^2 - 2ab + 4b^2)$$

$$= \underline{a^2 + 2a^2} + \underline{3ab - 2ab} + \underline{b^2 + 4b^2}$$

$$= 3a^2 + ab + 5b^2$$

Is the above expression a polynomial ?

$$A - B = (a^2 + 3ab + b^2) - (2a^2 - 2ab + 4b^2)$$

$$= a^2 + 3ab + b^2 - 2a^2 + 2ab - 4b^2$$

$$= \underline{a^2 - 2a^2} + \underline{3ab + 2ab} + \underline{b^2 - 4b^2}$$

$$= -a^2 + 5ab - 3b^2$$

Is the expression $A - B$ a polynomial ?

From the above examples, we can say that the sum or difference of any two polynomials is always a polynomial.

4. If $A = x^2 - 5x + 6$ and $B = x^2 + 2$ express $A - 2B$ in its simplest form.

$$A - 2B = x^2 - 5x + 6 - 2(x^2 + 2)$$

$$= x^2 - 5x + 6 - 2x^2 - 4$$

$$= \underline{x^2 - 2x^2} - 5x + \underline{6 - 4} \qquad = -x^2 - 5x + 2$$

Exercise 8.1

1. Add the following polynomials.

(1) $3x^2 + 5x + 10;\ 2x^2 + x + 2$

(2) $4a^2 - 5a + 6;\ a^2 + 2a - 5$

(3) $5a^2b^2 - 7ab^3 + 2b^4\ ;\ 4a^2b^2 + 3ab^3 + b^4$

(4) $a^4 - b^4\ ;\ a^4 + b^4$

(5) $m^2 - 5mn + 7n^2\ ;\ -m^2 + 5mn - 7n^2$

(6) $x^3 - 5x^2 + 7x + 8\ ;\ 4x^3 + 2x^2 - 7x$

(7) $x^4 + x^3 + x^2 - x + 1\ ;\ x^8 - x^2$

(8) $\frac{1}{2}x^2 - \frac{5}{3}x + 7\ ;\ \frac{1}{5}x^2 + \frac{1}{2}x - 5$

(9) $10x^2 + 6x + 9\ ;\ 0$

(10) $x^2 - 5x + 6\ ;\ -x^2 + 5x - 6$

2. Subtract the second polynomial from the first.

(1) $3x^3 - 5x^2 + 7x + 6$; $2x^3 + x^2 - 5x + 2$

(2) $7a^2 + 5a - 6$; $2a - 7$

(3) $5b^4 - 4b^2 - 3b$; $3b + 5b^4$

(4) $a^4 - 3a^3b + 5a^2b^2 + 7ab^3 - 4b^4$;

$-a^4 + 5a^3b - 4a^2b^2 + 6ab^3 - b^4$

(5) $\frac{2}{3}x^2 - \frac{1}{3}x - 5$; $\frac{1}{3}x^2 + \frac{2}{3}x + \frac{5}{3}$

(6) $a^3 - 5a^2 + 7$; $5a - 7a^2 + a^3$

(7) $m^2 - mn + n^2$; $m^2 - n^2$

(8) 0; $x^2 - y^2 + 5xy$

(9) $x^3 - 5x + 6$; 0

(10) $p^2 - 4p + 7$; $p^2 - 4p + 7$

3. Simplify

(1) $(4x^2 - 5x + 7) - (3x^2 + 2x + 1) + (-x^2 + x - 5)$

(2) $(a^2 + 5a - 6) - (2a^2 + 2a - 3) + (a^2 - 7a + 9)$

(3) $(m^2 - 5m + 7) + (4m^2 + m - 2) - (3m^2 - 2m)$

(4) $-(x^2 + 5x - 6) + (x^3 + x^2 + 5x) - (x^3 + 4)$

(5) $(a + b) + 2(b + c) + 3(c + a)$

(6) $5(x - 3) + 7(x^2 - x + 2) - 3(x^2 + 5x + 4)$

(7) $(y^2 - 5y + 6) - 2(y^2 - 2)$

(8) $2(x + y + z) - 3(x - y + z) + (x - y - z)$

4. Find the polynomial which when added to the sum of the polynomials $x^2 - 5x + 7$ and $2x^2 + 4$ will give the sum $5x^2 + 4x + 3$.

5. Find the polynomial which when subtracted from the polynomial $a^2 - 2a + 5$ will give the polynomial $2a^2 + 5$.

6. Subtract the polynomial $m^2 + 2m - 3$ from the polynomial $3m^2 - 5m + 7$ and add to the remainder the polynomial $-2m^2 + 2m + 2$.

7. If $A = 5x^2 - 7x + 6$, $B = 2x^2 + 7$ and $C = x^2 + 2x - 3$, express the polynomial D in the following examples in its simplest form,

(1) $A + B = D$ (2) $B - C = D$ (3) $A - 2B = D$

(4) $A + D = C$ (5) $D - B = A$ (6) $A - D = B$

(7) $A + B = C + D$ (8) $A + D = A$ (9) $A + B = A + D$

(10) $A - C = A - D$ (11) $A + D = 0$ (12) $A - D = 0$

§ 8.2 Degree of a polynomial

Every monomial is a polynomial. First let us consider a polynomial having only one term. (monomial).

In the monomial $6a^2b^3$ index of a is 2 and that of b is 3. The sum of the indices of the variables a and b in this monomial is 5. Hence degree of the monomial $6a^2b^3$ is 5.

The degree of any monomial is the number showing the sum of the indices of all the variables in it. You have learnt that the number 7 is a monomial. 7 can be written as $7x^0$, hence degree of the monomial 7 is zero.

The monomial '0' can be written as ox°, ox^5, om^8. Hence it is not possible to determine its degree. Therefore we say that the monomial '0' has no degree.

The values of the monomials like $5x, -6a^3b, -3m^4$ change with the values of the variables. But the values of the monomials like $7, -\frac{5}{3}, 0, -10$ do not change, hence they are called 'constant monomial.' The constant monomial zero has no degree and the degree of the non-zero constant monomial is zero.

The number $-\frac{7}{5}$ in the monomial $-\frac{7}{5}x^2y$ is called as the coefficient of the monomial.

Exercise 8.2

1. State the degree of each of the following monomials.

(1) $4x^2$ (2) $\frac{9}{13}a$ (3) $-\frac{5}{9}$ (4) x^2y (5) 0 (6) $-m^2n$ (7) $\frac{1}{2}x^n$ (n is positive integer) (8) x^my^n (m and n are positive integers)

2. State the coefficient of each of the following monomials.

(1) $5x^3$ (2) $-4y^2$ (3) m^2 (4) $5x$ (5) $-a$

3. Fill in the blanks.

(1) The coefficient of the monomial $-5x^3$ is ______ and its degree is ____

(2) Of $-x^3$ and x^{-3}, ... is not a monomial because ....

(3) Of x^2 and $x^{\frac{1}{2}}$, ... is not a monomial because ....

(4) .... monomial has no degree.

(5) Of $7x^3$, $7x$ and 7, the degree. of the monomial.... is zero.

(6) The degree of the monomial $4a^3b$ is ....

(7) The degree of the monomial .... is 1 and its coefficient is -5 (use m for the variable).

We now know how the degree of a polynomial having one term is determined.

The polynomial is expressed as a sum or difference of monomials. These monomials are called terms of the polynomial. The highest of the degrees of the terms of the polynomial is called the degree of that polynomial.

In the polynomial $3x^3y - xy + 7x^4 - 5xy^4$ the degrees of the terms are 4, 2, 4 and 5 respectively. Out of these, the highest degree is 5. Hence the degree of the above polynomial is 5.

In the discussion to follow we shall mainly consider polynomials in only one variable.

§ 8.3 Cofficient Form of a Polynomial

Generally the polynomials are written in the descending orders of the indices of the variable. The polynomial $5x^3 - 4x + x^6 + 7$ can be written in the descending order of the indices of the variable x as $x^6 + 5x^3 - 4x + 7$. In this polynomial some powers of x are absent. The terms containing the powers x^5, x^4 and x^2 are missing. This means that the coefficients of these terms are zero. Now inserting these missing terms, let us express the polynomial in descending order of the indices.

$x^6 + ox^5 + ox^4 + 5x^3 + ox^2 - 4x + 7$ ——— (I)

Now there are 7 terms in the polynomial and its degree is 6. We can express this polynomial as (1, 0, 0, 5, 0, -4, 7) by writing coefficients of each of its terms. This form of expressing a polynomial is known as the coefficient form of the polynomial and the form (see I) where the symbols (x, y etc.) are used is known as the index form of the polynomial.

Let us write the polynomial $x^4 - 2x$ in the coefficient form.

Here terms containing the powers x^3, x^2 and the constant term are missing. Let us write the polynomial, inserting the missing terms and arrange in the descending order of indices.

$x^4 + 0x^3 + 0x^2 - 2x + 0$

Now it can be expressed in the coefficient form as

(1, 0, 0, − 2, 0).

Here we find that the number of terms in the polynomial is 5 and its degree is 4. Which means that the number of coefficients is 1 more than its degree.

Now let us see, what is the relation between the degree and the number of coefficients of the non-zero constant polynomial. The degree of the non-zero constant polynomial is zero and the number of coefficients is 1. Therefore the number of coefficients in the non-zero constant polynomial is also 1 more than its degree.

Examples–

1. Express the following polynomial given in the coefficient form, in the index form. (4, 0, 0, − 7, 5)

For writing in the index form, we will have to determine its degree. The number of coefficients is 5, hence the degree of the polynomial is $5 - 1 = 4$. By using the variable x, we can express the polynomial in the index form, as $4x^4 + 0x^3 + 0x^2 - 7x + 5$. Deleting terms with zero coefficients the above polynomial is written as $4x^4 - 7x + 5$.

2. Express the polynomial (1, − 1, 0) given in the coefficient form, in the index form.

Here the number of coefficients is 3. Hence the degree of the polynomial is 2.

$\therefore$ $1x^2 - 1x + 0$ i.e. $x^2 - x$ is the required index form of the polynomial.

3. Express the polynomial $5x^4$ in the coefficient form.

$5x^4 = 5x^4 + 0x^3 + 0x^2 + 0x + 0$

$\therefore$ (5, 0, 0, 0, 0) is the required coefficient form of the polynomial.

4. Express the polynomial $ax^3 + bx^2 + 7$ in the coefficient form.

$ax^3 + bx^2 + 7 = ax^3 + bx^2 + 0x + 7$

$\therefore$ $(a, b, 0, 7)$ is the required coefficient form of the polynomial.

5. If $A = ax^4 + bx^3 + cx^2 + dx + e$ and $B = 5x^4 + 3x^3 + 5$ are equal polynomials then find the values of a, b, c, d and e.

In order that A and B are equal polynomials the coefficients of like powers in both polynomials should be equal. In the polynomial B the powers x^2 and x are the missing terms, which means that coefficients of x^2 and x are zero.

$\therefore$ $a = 5$, $b = 3$, $c = 0$, $d = 0$ and $e = 5$,

Exercise 8.3

1. State the degrees of the following polynomials and express them in the coefficient form,

(1) $3x^2 - 10x + 7$
(2) $y^4 + 7y^3 + 7y - 6$
(3) $x^4 - 5x$
(4) $7 - 5a + a^4$
(5) $-7y^5$
(6) -5
(7) $x^3 - px + q$
(8) x
(9) $ax^2 + b$
(10) $-x$
(11) $ax^3 - dx$
(12) $ax - c$

2. State the degrees of the following polynomials and express them in the index form.

(1) $(1, 10, -5)$
(2) $(5, 0, 0, 0, -7)$
(3) $(4, -3)$
(4) $(2, 0, 0, 0, 0)$
(5) (a, b, c, d)
(6) $(a, 0, 0, b)$

3. If the polynomials $ax^3 + bx^2 + cx + d$ and $4x^3 - 5x^2 + 7x - 6$ are equal, find values of a, b, c and d.

4. If the polynomials $ax^2 - bx + c$ and $7x^2 + 5x - 4$ are equal, find values of a, b, c and $a + b + c$.

§ 8.4 Multiplication of polynomials

1. Multiply the polynomials $4x^2 - 5x + 7$ and $2x + 3$.

$$\begin{aligned}
& (4x^2 - 5x + 7) \times (2x + 3) \\
&= (4x^2 - 5x + 7) \times 2x + (4x^2 - 5x + 7) \times 3 \text{ (removing brackets)} \\
&= 8x^3 - 10x^2 + 14x + 12x^2 - 15x + 21 \text{ (removing brackets)} \\
&= 8x^3 \underline{- 10x^2 + 12x^2} + \underline{14x - 15x} + 21 \\
&= 8x^3 + 2x^2 - x + 21
\end{aligned}$$

Following is another method for multiplication.

$$\begin{array}{l}
4x^2 - 5x + 7 \\
\times \quad 2x + 3 \\
\hline
8x^3 - 10x^2 + 14x \quad \text{............ (multiplying by } 2x\text{)} \\
\qquad\quad 12x^2 - 15x + 21 \quad \text{......... (multiplying by 3)} \\
\hline
8x^3 + 2x^2 - x + 21 \quad \text{........... (Adding)}
\end{array}$$

2. If $A = 4y + 5y^3 - 7$ and $B = y^2 - 2y + 3$, express $A \times B$ in its simplest form.

Let us insert the missing power y^2 in the polynomial A and rewrite A in the descending order of the indices and multiply it by the polynomial B,

$$\begin{array}{l}
5y^3 + 0y^2 + 4y - 7 \\
\times \qquad y^2 - 2y + 3 \\
\hline
5y^5 + 0y^4 + 4y^3 - 7y^2 \text{.......... (multiplying by } y^2\text{)} \\
\qquad -10y^4 + 0y^3 - 8y^2 + 14y \text{..... (multiplying by } -2y\text{)} \\
\qquad\qquad\qquad 15y^3 + 0y^2 + 12y - 21 \text{ (multiplying by 3)} \\
\hline
5y^5 - 10y^4 + 19y^3 - 15y^2 + 26y - 21 \text{ (Adding)}
\end{array}$$

Is the expression $A \times B$ above a polynomial ?
What is the relation between the degree of the polynomial $A \times B$ and the degrees of the polynomials A and B ?

The degree of the polynomial A is 3 and the degree of the polynomial B is 2, while the degree of the polynomial $A \times B$ is

5, $(3 + 2 = 5)$

We can observe that the same relation holds between the degrees of the two polynomials and the degree of their product in ex.1 also.

We can generalise this as –

The product of two polynomials of degrees m and n is a polynomial of degree $m + n$. (This obviously holds true only for non-zero polynomials.)

Exercise 8.4

1. Multiply the polynomials in the following examples and state the degree of their products.

(1) x^2+5x+2 ; $x+3$ (2) $5x^2-2x+3$; $x-2$

(3) a^3+4a^2 ; a^2-5a+7 (4) m^4-1 ; m^2+5

(5) y^2-1 ; y^2+1 (6) x^2-x+1 ; $x+1$

(7) $5x^3-7x+6$; $x-3$ (8) x^2-2 ; x^3+6x+5

(9) $4y^2-6y+7$; y^2-y-2 (10) ax^2+bx+c ; $x-p$

2. Complete the following

Polynomial A	Polynomial B	Polynomial $A\times B$
x^2+5x-7	1	
y^3-3y+4	1	
a^2-7a+8	1	
x^3-5x^2+7x-9	1	
$2y^2-5y-3$	0	
m^3-7m	0	
$a^2+2ab+b^2$	0	
a^3+b^3	0	

Use the above table and fill in the blanks.

(1) If any polynomial A is multiplied by the constant polynomial 1 the result is polynomial ......

(2) If any polynomial A is multiplied by the '0' polynomial, the result is .... polynomial.

§ 8.5 Division of Polynomials

We have seen how to add, subtract and multiply two polynomials. Now we shall see how to divide one polynomial by another polynomial.

1. Divide the polynomial x^2+5x+6 by the polynomial $x+2$.

$$
\begin{array}{rll}
 & x+3 & \\
x+2\,\big) & \overline{x^2+5x+6} & \\
 & \underline{\underset{-}{x^2}\underset{-}{+}2x} & \ldots\ldots\ldots\ (\text{product of } x \text{ and } x+2) \\
 & 3x+6 & \ldots\ldots\ldots\ (\text{first remainder}) \\
 & \underline{\underset{-}{3x}\underset{-}{+}6} & \ldots\ldots\ldots\ (\text{product of } 3 \text{ and } x+2) \\
 & 0 & \ldots\ldots\ldots\ (\text{second remainder})
\end{array}
$$

Thus, if x^2+5x+6 is divided by $x+2$ the quotient is $x+3$ and the remainder is zero. We can express this in the form.

Dividend = Divisor × Quotient + Remainder as

$x^2+5x+6 = (x+2)(x+3)+0$

In the above example as the remainder is zero we can express this as

$(x^2+5x+6)\div(x+2) = x+3$ (*a* polynomial)

2. If $A = 3-x^3+5x$ and $B = 2+x$, divide the polynomial A by the polynomial B and express the dividend as – -
Dividend = Divisor × Quotient + Remainder.

Let us insert the missing powers in the dividend and the divisor in descending order of indices.

$$
\begin{array}{rll}
 & -x^2+2x+1 & \\
x+2\,\big) & \overline{-x^3+0x^2+5x+3} & \\
 & \underline{\underset{+}{-x^3}\ \underset{+}{-}\ 2x^2} & \ldots\ldots\ldots\ldots\ldots\ldots\ (\text{product of } -x^2 \text{ and } x+2) \\
 & 2x^2+5x+3 & \ldots\ldots\ldots\ (\text{first remainder}) \\
 & \underline{\underset{-}{2x^2}\underset{-}{+}4x} & \ldots\ldots\ldots\ldots\ (\text{product of } 2x \text{ and } x+2) \\
 & x+3 & \ldots\ldots\ (\text{second remainder}) \\
 & \underline{x+2} & \ldots\ldots\ (\text{product of } 1 \text{ and } x+2) \\
 & 1 & \ldots\ldots\ (\text{third remainder})
\end{array}
$$

As the degree of remainder i.e. polynomial 1, is less than the degree of the divisor i. e. polynomial $x+2$, the division connot be performed any further.

In this example, dividend $= 3-x^3+5x$, divisor $= 2+x$, quotient = $-x^2 + 2x + 1$ and the remainder = 1

$\therefore\ 3 - x^3 + 5x = (2 + x)(-x^2 + 2x + 1) + 1$

Here the remainder is not zero but the constant polynomial 1.

$$\therefore\ (3 - x^3 + 5x) \div (2 + x) = (-x^2 + 2x + 1) + \frac{1}{2 + x}$$

which means that in this example $A \div B$ is not a polynomial. (It is clear from the right hand side.)

3. Divide $4x^3 + 15x - 7$ by $2x^2 - x + 5$ and express it in the form Dividend = Divisor × Quotient + Remainder

$$\begin{array}{rl}
 & \qquad 2x + 1 \\ \cline{2-2}
2x^2 - x + 5 \;\big) & 4x^3 + 0x^2 + 15x - 7 \\
 & 4x^3 - 2x^2 + 10x \\
 & \underline{- \quad\;\; + \quad\;\; - \qquad\qquad} \\
 & \qquad 2x^2 + 5x - 7 \quad \ldots\ldots \text{ (first remainder)} \\
 & \qquad 2x^2 - x + 5 \\
 & \qquad \underline{- \quad + \quad - \;\;} \\
 & \qquad\qquad 6x - 12 \quad \ldots\ldots \text{ (Second remainder)}
\end{array}$$

As the degree of the remainder is less than that of the divisor, division cannot be performed any further.

$\therefore\ 4x^3 + 15x - 7 = (2x^2 - x + 5)(2x + 1) + (6x - 12)$

In this example the degrees of the dividend, divisor, quotient and the remainder are 3, 2, 1 and 1 respectively.

4. Divide the polynomial $x + 4$ by the polynomial $x^2 - 2x + 5$.

In this example the degree of the dividend is less than the degree of the divisor. Hence division cannot be carried out or in other words we can say that the quotient is zero and the remainder is same as the polynomial in the dividend,

$$\begin{array}{rl}
 & \quad 0 \\ \cline{2-2}
x^2 - 2x + 5 \;\big) & x + 4 \\
 & \underline{- \;\; 0 \quad} \\
 & x + 4
\end{array}$$

From the above examples we observe that

(1) We cannot divide a polynomial by another polynomial whose degree is greater than the degree of the polynomial in the dividend.

(2) The remainder is either a zero polynomial or a polynomial of degree less than that of the divisor.

(3) The degree of the quotient is equal to the degree of the dividend minus the degree of the divisor.

Exercise 8.5

Divide first polynomial by the second and express the result in the form.

divident = divisor × qnotient + remainder

(1) $8x^2 + 4x$; $x + 2$
(2) $y^3 - 9$; $y + 3$
(3) $m^3 + 64$; $m + 4$
(4) $2a^2 + 3a + 1$; $a + 2$
(5) $x^3 - 4x^2 + 5x - 2$; $x - 1$
(6) $x^2 - x - 2$; $x - 1$
(7) $4x^2 + 16x$; $4x + 12$
(8) $2x^4 + x^3 - 19x^2 + 5x + 15$; $x^2 - 7$
(9) $125y^3 + 1$; $5y + 1$
(10) $x^4 + x^2 + 1$; $x^2 + x + 1$
(11) $x - 10 + 3x^2$; $x + 2$
(12) $y^3 + 10y^2 - 48y + 1$; $y^2 - 5y + 7$
(13) $x^2 - a^2$; $x + a$
(14) $x^3 + a^3$; $x + a$
(15) $ax^2 + bx + c$; $x - p$
(16) $ax^2 + bx + c$; $x + p$

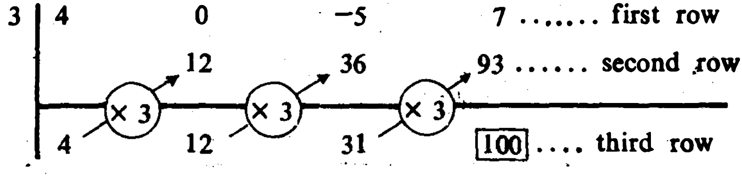
§ 8.6 Synthetic Division

You have learnt how to divide one polynomial by another, when they are expressed in the index form. Now we shall study an easier method of division when the divisor is of the form $x \pm a$. This method of division is called 'synthetic division'

1. Divide the polynomial $4x^3 - 5x + 7$ by $x - 3$ and express the result in the form –

dividend = divisor × quotient + remainder.

Let us write the dividend in the coefficient form. In this example the divisor is $x - 3$, so for the purpose of synthetic division we consider the divisor as 3 which is the opposite of -3.



Explanation :

(1) Write the dividend in the coefficient form in the first row.

(2) Divisor is $(x - 3)$ ∴ In place of the divisor write 3 whice is opposite of $- 3$.

(3) Draw a vertical line between the divisor and the dividend and draw a horizontal line below the dividend leaving some space for the second row.

(4) Below the horizontal line, in the third row, write the first coefficient 4.

(5) Multiply the first coefficient 4 in the third row by 3, the number in place of the divisor and write the product 12, below the second coefficient 0, in the second row. Write the sum of the second coefficients (here 0 and 12) in the first and the second row in the third row.

(6) Write the product of 12 (from the third row) and 3 (the number in place of the divisor) i. e 36 below $- 5$ in the second row. Write the sum of $- 5$ and 36 i. e 31 in the third row.

(7) Write the product of 31 (from the third row) and 3 (the number in place of the divisor) i. e. 93 below 7, in the second row. Write the sum of 7 and 93. i. e. 100 in the third row

(8) The last number 100 in the third row is the remainder and rest of the coefficients in the order give the quotient (4, 12, 31) in the coefficient form.

∴ quotient $= 4x^2 + 12x + 31$ and remainder $= 100$

Hence $4x^3 - 5x + 7 = (x - 3) (4x^2 + 12x + 31) + 100$

2. Divide $5x^4 - 6x^3 + 5x - 18$ by $x - 2$.

Let us write the dividend in the coefficient form. The divisor is $x - 2$. Hence in place of the divisor we shall write 2. Which is opposite of $- 2$.

$$\begin{array}{r|rrrrr}
2 & 5 & -6 & 0 & 5 & -18 \\
 & & 10 & 8 & 16 & 42 \\
\hline
 & 5 & 4 & 8 & 21 & \boxed{24}
\end{array}$$

∴ quotient $= (5, 4, 8, 21) = 5x^3 + 4x^2 + 8x + 21$ and remainder $= 24$

$\therefore\ 5x^4 - 6x^3 + 5x - 18 = (x-2)(5x^3 + 4x^2 + 8x + 21) + 24$

3. Divide $7y^3 - 5y^5 + 4y - 23$ by $y + 2$

Let us write the dividend in the coefficient form. The divisor is $y + 2$ $\therefore$ in place of the divisor we write -2 which is opposite of 2.

$$\begin{array}{r|rrrrrr} -2 & -5 & 0 & 7 & 0 & 4 & -23 \\ & & 10 & -20 & 26 & -52 & 96 \\ \hline & -5 & 10 & -13 & 26 & -48 & \boxed{73} \end{array}$$

$\therefore$ quotient $= -5y^4 + 10y^3 - 13y^2 + 26y - 48$; remainder $= 73$

$\therefore\ 7y^3 - 5y^5 + 4y - 23 = (y+2)(-5y^4 + 10y^3 - 13y^2 + 26y - 48) + 73.$

4. Divide $x^4 - 81$ by $x + 3$

$x^4 - 81 = x^4 + 0x^3 + 0x^2 + 0x - 81$

Let us write -3 in place of the divisor (here divisor is $x + 3$)

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & 0 & -81 \\ & & -3 & 9 & -27 & 81 \\ \hline & 1 & -3 & 9 & -27 & \boxed{0} \end{array}$$

$\therefore$ quotient $= x^3 - 3x^2 + 9x - 27$; remainder $= 0$

$\therefore\ x^4 - 81 = (x+3)(x^3 - 3x^2 + 9x - 27)$

5, Divide $ay^2 + by + c$ by $y - p$

As the divisor is $y - p$. write p in place of the divisor.

$$\begin{array}{r|lll} p & a & b & c \\ & & ap & ap^2 + bp \\ \hline & a & ap + b & \boxed{ap^2 + bp + c} \end{array}$$

$\therefore$ quotient $= ay + (ap + b)$; remainder $= ap^2 + bp + c$

$\therefore\ ay^2 + by + c = (y - p)\,[ay + (ap + b)] + ap^2 \times bp + c$

Exercise 8.6

Use synthetic division method for performing following divisions. Write the result in the form dividend = divisor × quotient × remainder.

(1) $(x^3 - 5x^2 + 7x + 2) \div (x - 1)$ (2) $(x^4 - 5x^2 + 3) \div (x - 1)$

(3) $(y^3 + 7y^2 + 5y + 3) \div (y + 1)$ (4) $(y^4 - 7y^2 + 2) \div (y - 1)$

(5) $(x^4 - 7x^2 + 11x - 5) \div (x-1)$

(6) $(y^3 + 7y^2 + 9y + 3) \div (y+1)$

(7) $(x^4 - 16) \div (x-2)$

(8) $(y^4 - 16) \div (y+2)$

(9) $(x^3 + 125) \div (x+5)$

(10) $(x^3 - 7x + 3) \div (x-3)$

(11) $(4x^4 - 5x^2 + 15) \div (x-3)$

(12) $(5y^4 - 2y^3 + 7y - 5) \div (y-2)$

(13) $(x^3 + 0x^2 + 0x - a^3) \div (x-a)$

(14) $(x^3 + a^3) \div (x+a)$

(15) $(ax^2 + bx + c) \div (x-p)$

(16) $(ax^2 + bx + c) \div (x+p)$

§ 8.7 Divisibility in Polynomials

When one polynomial is divided by another polynomial, we have seen that

Dividend = divisor × quotient + remainder where the remainder is either a zero polynomial or a polynomial of degree less than the degree of the divisor.

$$0 \leq \text{degree of the remainder} < \text{degree of divisor.}$$

If dividend $= P$, divisor $= A$, quotient $= Q$ and remainder $= R$ then $P = A \times Q + R$ where (1) R is a zero polynomial or (2) degree of R is less than degree of A.

Division cannot be carried out if the degree of the divisor is greater than the degree of the dividend.

If $R = 0$ (remainder is a zero polynomial) we get $P = A \times Q$. In this case P is said to be divisible by A and if $R \neq 0$, then P is said to be not divisible by A.

Examples :

1. If $P = 7x^3 - 5x^2 + 6x - 4$ and $A = x + 2$, find whether P is divisible by A.

Let us find the remainder by synthetic division.

$$\begin{array}{r|rrrr} -2 & 7 & -5 & 6 & -4 \\ & & -14 & 38 & -88 \\ \hline & 7 & -19 & 44 & \boxed{-92} \end{array}$$

Here remainder = −92 i.e. non-zero

$\therefore$ P is not divisible by A.

2. Determine whether $4x^4 - 5x^2 + 6x - 5$ is divisible by $x - 1$

$$\begin{array}{r|rrrrr} 1 & 4 & 0 & -5 & 6 & -5 \\ & & 4 & 4 & -1 & 5 \\ \hline & 4 & 4 & -1 & 5 & \boxed{0} \end{array}$$

Here remainder is zero.

$\therefore$ $4x^4 - 5x^2 + 6x - 5$ is divisible by $x - 1$.

3. Is $6x^3 - x^2 - 22x + 15$ divisible by $3x^2 + 4x - 5$?

In this example the divisor is not of the form $x \pm a$. Hence we cannot perform synthetic division. Hence we shall find the remainder by the usual method of division.

$$\begin{array}{rl} & \phantom{6x^3 - {}} 2x - 3 \\ 3x^2 + 4x - 5 \,\big) & \overline{6x^3 - x^2 - 22x + 15} \\ & 6x^3 + 8x^2 - 10x \\ & \underline{- \quad\;\; - \qquad\;\; + \qquad\qquad} \\ & - 9x^2 - 12x + 15 \\ & - 9x^2 - 12x + 15 \\ & \underline{ + \qquad\; + \qquad\; -} \\ & \phantom{6x^3 - 9x^2 -{}} 0 \end{array}$$

Remainder is zero.

$\therefore$ $6x^3 - x^2 - 22x + 15$ is divisible by $3x^2 + 4x - 5$

Exercise 8.7

By synthetic division decide whether first polynomial is divisible by the second,

(1) $x^2 - x - 6$; $x - 3$

(2) $x^2 + 6x + 8$; $x + 2$

(3) $y^3 - 5y^2 + 6y - 2$; $y - 1$

(4) $m^4 - 81$; $m - 3$

(5) $x^3 - 5x^2 + 6x - 28$; $x - 4$

(6) $2m^3 + 7m^2 + 6m + 1$; $m + 1$

(7) $4y^3 - y^2 - 29y + 30$; $y + 3$

(8) $x^4 - 13x^2 + 10x + 6$; $x - 3$

(9) $p^3 - 3p^2 + 2p - 8$; $p + 4$

(10) $y^3 + 64$; $y + 4$

(11) $x^3 + 3ax^2 + 3a^2x + a^3$; $x + a$

(12) $x^3 - a^3$; $x - a$

§ 8:8 Value of the Polynomial

You have learnt how to find the value of the polynomial when value of the variable is given.

Examples :—

1. If $P = x^3 - 5x + 7$, find the values of :

(1) $P \big|_{x=3}$, (2) $P \big|_{x=-4}$ (3) $P \big|_{x=o}$, (4) $P \big|_{x=a}$.

$$P = x^3 - 5x + 7$$

$\therefore$ (1) $P \big|_{x=3} = (3)^3 - 5(3) + 7 = 27 - 15 + 7 = 19$

(2) $P \big|_{x=-4}, = (-4)^3 - 5(-4) + 7 = -64 + 20 + 7 = -37$

(3) $P \big|_{x=o} = (0)^3 - 5(0) + 7 = 7$

(4) $P \big|_{x=a} = (a)^3 - 5(a) + 7 = a^3 - 5a + 7$

2. If $A = y^3 - 7y^2 + 8$. find the values of :

(1) $A \big|_{y=2}$, (2) $A \big|_{y=o}$, (3) $A \big|_{y=-1}$ and

(4) $A \big|_{y=6}$

$$A = y^3 - 7y^2 + 8$$

$\therefore$ (1) $A \big|_{y=2} = (2)^3 - 7(2)^2 + 8 = 8 - 28 + 8 = -12$

(2) $A \big|_{y=0} = (0)^3 - 7(0)^2 + 8 = 8$

(3) $A \big|_{y=-1} = (-1)^3 - 7(-1)^2 + 8 = -1 - 7 + 8 = 0$

(4) $A\Big|_{y=b} = (b)^3 - 7(b)^2 + 8 = b^3 - 7b^2 + 8$

3. If the value of the polynomial $2x^2 + bx + 5$ when $x = 2$ is 29, find b.

Let $P = 2x^2 + bx + 5$

$\therefore P\Big|_{x=2} = 2(2)^2 + b(2) + 5$

$\therefore 29 = 8 + 2b + 5$

$\therefore 29 - 13 = 2b$

$\therefore 16 = 2b$

$\therefore 8 = b$

$\therefore b = 8$

4. If the value of the polynomial $ax^3 - 6x^2 + 5x + b$ when $x = -1$ is -7 and when $x = 2$ it is 8 find a and b.

Let $P = ax^3 - 6x^2 + 5x + b$

$\therefore P\Big|_{x=-1} = a(-1)^3 - 6(-1)^2 + 5(-1) + b$

$\therefore -7 = -a - 6 - 5 + b$

$\therefore -7 = -a + b - 11$

$\therefore a - b = -4$ .... (I)

Similarly $P\Big|_{x=2} = a(2)^3 - 6(2)^2 + 5(2) + b$

$\therefore 8 = 8a - 24 + 10 + b$

$\therefore 8a + b = 22$ .... (II)

Solve the simultaneous equations (I) and (II) and obtain values of a and b as $a = 2$ and $b = 6$

Exercise 8.8

1. Find the values of the polynomial $3x^2 + 5x - 4$ when (1) $x=0$, (2) $x=1$, (3) $x=-2$, (4) $x=a$, (5) $x=-p$.

2. Find the values of the polynomial $y^3 + 2y - 3$ when (1) $y=-1$ (2) $y=2$, (3) $y=\frac{1}{2}$, (4) $y=-2a$, (5) $y=(a+b)$.

3. Find the values of the polynomial $x^3 - 3ax^2 + 3a^2x - a^3$ when (1) $x = a$ and (2) $x = -a$

4. Find the value of the polynomial $px^2 + qx + r$ when $x = a$.

5. Find the value of the poiynomial $2ax^2 - 3ax + 5x + a^2$ when $x = -2a$.

6. Find the value of the polynomial $y^2 - 2y$ when $y = a + \frac{1}{a}$

7. If the value of the polynomial $x^3 - 9x^2 + cx - 8$ when $x = -1$ is -10, find c.

8. If the value of the polynomial $x^3 + ax^2 + bx + 7$ when $x = 1$ is 3, and when $x = -2$ it is 9, find a and b.

9. If the value of the polynomial $2x^3 + px^2 + qx + 10$ when $x = 1$ is 0. and when $x = -2$ it is -2 find p and q.

10. If the value of the polynomial $4x^3 + bx^2 - 5x + c$ when $x = 0$ is 7, and when $x = 1$ it is 6. find b and c.

§ 8.9 Remainder Theorem :

Divide $P = 4x^3 - 5x^2 + 7x - 10$ by $x - 2$ and express the result in the form dividend = divisor × quotient + remainder.

$$\begin{array}{c|cccc} 2 & 4 & -5 & 7 & -10 \\ & & 8 & 6 & 26 \\ \hline & 4 & 3 & 13 & \boxed{16} \end{array}$$

Here quotient $= 4x^2 + 3x + 13$ and remainder $= 16$

$\therefore \; 4x^3 - 5x^2 + 7x - 10 = (x - 2)(4x^2 + 3x + 13) + 16$

Let us find $P \Big|_{x = 2}$

$$P \Big|_{x=2} = 4(2)^3 - 5(2)^2 + 7(2) - 10$$

$$= 32 - 20 + 14 - 10$$

$$= 16$$

Thus the value of the polynomial P when $x = 2$ is the same as the remainder when P is divided by $x - 2$

Can this be proved ?

$$4x^3 - 5x^2 + 7x - 10 = (x-2)\left(4x^2 + 3x + 13\right) + 16$$

In this equality if x is replaced by 2, we get 16 on the right hand side. But it is the remainder when the polynomial is divided by $x - 2$.

$$\text{Thus } P\Big|_{x=2} = R,$$

where R is the remainder when P is divided by $x - 2$. This is known as the ' **Remainder** Theorem. '

Remainder Theorem :– If a polynomial P in x is divided by $x - a$, then the remainder is the value of the polynomial P when $x = a$.

Examples :

1. Find the remainder when the polynomial $P = x^8 - 12x^2 + 7x + 9$ is divided by $x - 2$.

In this example the divisor $= x - 2$. Therefore we take $x - 2 = 0$ i.e. $x = 2$.

$\therefore$ By Remainder Theorem the required remainder $= P\Big|_{x=2}$

$$P = x^3 - 12x^2 + 7(x) + 9$$

$$\therefore P\Big|_{x=2} = (2)^3 - 12(2)^2 + 7(2) + 9$$

$$= 8 - 48 + 14 + 9$$

$$= -17$$

$\therefore$ required remainder $= -17$

The remainder will be -17 even if it is calculated by synthetic division.

2. If $3x^4 + 5x^3 + px + 17$ is divided by $x + 3$ the remainder is 53. Find p by Remainder Theorem.

Here divisor $x + 3$ $\therefore$ equating $x + 3$ to zero we get $x = -3$.

$$A = 3x^4 + 5x^3 + px + 17$$

$$\therefore \left. A \right|_{x=-3} = 3(-3)^4 + 5(-3)^3 + p(-3) + 17$$

$$\therefore 53 = 243 - 135 - 3p + 17$$

$$\therefore 3p = 125 - 53 = 72$$

$$\therefore p = 24$$

Exercise 8.9

1. Using Remainder Theorem find the remainder when the first polynomial is divided by the second polynomial. Check your answer by synthetic division,

(1) $x^2 + 5x + 6$; $x - 2$
(2) $x^2 + 5x - 6$; $x - 1$
(3) $y^2 - y - 12$; $y - 3$
(4) $3m^2 - 2m + 5$; $m+2$
(5) $x^2 - 7x + 4$; $x + 2$
(6) $x^2 - 7x + 12$; $x - 4$
(7) $x^3 - 8$; $x - 2$
(8) $y^3 - 2y^2 + 3y - 2$; $y - 1$
(9) $y^3 - 5y + 7$; $y + 1$
(10) $x^3 - x + 2$; $x + 2$

2. When $ax^2 + 5x + 6$ is divided by $x - 1$ the remainder is 13 Find a.

3. When $2x^2 + bx + 5$ is divided by $x+1$ the remainder is 4. Find b.

4. When $x^2 + x - 2a$ and $ax^2 - 3x$ are divided by $x - 2$ the remainder is same. Find a.

5. When $x^3 - 5x^2 + 13x + p$ is divided by $x - 2$ the remainder is 30. Find p.

6. When $4x^3 + ax + b$ is divided by $x - 1$ the remainder is -1 and when it is divided by $x+2$ the remainder is -52. Find a and b.

7. When $y^3 + 7y^2 + py - 26$ is divided by $y + 2$ the remainder is 2. Find p.

8. By Remainder Theorem, find the remainder when $P = x^2 - 5x + 4$ is divided by $x - 1$. Is P divisible by $x - 1$? If it is divisible, then express P as $P = (x - 1) \times Q$, using synthetic division method.

§ 8.10 Factor Theorem :

Using Remainder Theorem decide whether the polynomial $A = x^3 + 3x^2 + 4x + 4$ is divisible by $x + 2$. Hence state whether $x + 2$ is a factor of $x^3 + 3x^2 + 4x + 4$.

By Remainder Theorem, the remainder when A is divided by $x+2$ is $A\Big|_{x=-2}$

$$A = x^3 + 3x^2 + 4x + 4$$

$$\therefore \quad A\Big|_{x=-2} = (-2)^3 + 3(-2)^2 + 4(-2) + 4$$

$$= -8 + 12 - 8 + 4$$

$$= 0$$

As $A\Big|_{x=-2}$ is zero, the remainder when A is divided by $x+2$ is zero, which means that the polynomial A is divisible by $x+2$.

$\therefore$ $x+2$ is a factor of $A = x^3 + 3x^2 + 4x + 4$.

Factor Theorem :- $x - a$ is a factor of the polynomial P in x, if the value of P when $x = a$ is zero.

Examples :

1. Decide whether $x-1$ is a factor of $P = x^3 - 5x^2 + 6x - 2$.

As we have to find whether $x-1$ is a factor of P, we shall find $P\Big|_{x=1}$

If $P\Big|_{x=1} = 0$ then $x-1$ will be a factor of P.

$$P = x^3 - 5x^2 + 6x - 2$$

$$\therefore \quad P\Big|_{x=1} = (1)^3 - 5(1)^2 + 6(1) - 2$$

$$= 1 - 5 + 6 - 2 = 0$$

$\therefore$ $x-1$ is a factor of the polynomial P.

2. Decide whether $x-2$ is a factor of the polynomial $P = x^3 + x^2 - 5x - 2$. If $x-2$ is a factor of P then express P as $P = (x-2) \times Q$.

We want to see whether $x-2$ is a factor of P. Hence we shall find $P\Big|_{x=2}$

$$P = x^3 + x^2 - 5x - 2$$

$$\therefore \; P\Big|_{x=2} = (2)^3 + (2)^2 - 5(2) - 2$$

$$= 8 + 4 - 10 - 2$$

$$= 0$$

$$P\Big|_{x=2} = 0 \quad \therefore \; x - 2 \text{ is a factor of } P.$$

We want to express P as $(x - 2) \times Q$, hence we divide P by $x - 2$. By synthetic division–

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -5 & -2 \\ & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & \boxed{0} \end{array}$$

Here remainder $= 0$ and quotient $Q = x^2 + 3x + 1$

$\therefore \; x^3 + x^2 - 5x - 2 = (x - 2)\,(x^2 + 3x + 1).$

3. Using Remainder Theorem decide whether $(x + 3)$ is a factor of $A = x^3 + 2x^2 - 2x + 3$ and if it is a factor, express A as product of two factors.

For finding whether $x + 3$ is a factor of A we shall find $A\Big|_{x=-3}$

$$A = x^3 + 2x^2 - 2x + 3$$

$$\therefore \; A\Big|_{x=-3} = (-3)^3 + 2(-3)^2 - 2(-3) + 3$$

$$= -27 + 18 + 6 + 3 = 0$$

$$A\Big|_{x=-3} = 0 \quad \therefore \; x + 3 \text{ is a factor of } A$$

Let us find the second factor by synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -2 & 3 \\ & & -3 & 3 & -3 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

Here remainder $= 0$ and quotient $Q = x^2 - x + 1$

$\therefore\ x^3 + 2x^2 - 2x + 3 = (x+3)(x^2 - x + 1)$

4. Using Remainder Theorem find p if $x+2$ is a factor of $B = 3x^3 + 10^2 + 5x + p$. Express B as $(x+2) \times Q$.

$(x+2)$ is a factor of B. $\therefore\ B\big|_{x=-2} = 0$

$$B = 3x^3 + 10x^2 + 5x + p$$

$$\therefore\ B\big|_{x=-2} = 3(-2)^3 + 10(-2)^2 + 5(-2) + p$$

$$\therefore 0 = -24 + 40 - 10 + p$$

$\therefore\ P = -6$

$\therefore\ B = 3x^3 + 10x^2 + 5x - 6$

Let us find Q by synthetic division.

$$\begin{array}{r|rrrr} -2 & 3 & 10 & 5 & -6 \\ & & -6 & -8 & 6 \\ \hline & 3 & 4 & -3 & \boxed{0} \end{array}$$

$\therefore\ Q = 3x^2 + 4x - 3$

$\therefore\ 3x^3 + 10x^2 + 5x - 6 = (x+2)(3x^2 + 4x - 3)$

[Same example can be solved conveniently using synthetic division. As $(x+2)$ is a factor of B, the remainder when B is divided by $x+2$ must be zero.

$$\begin{array}{r|rrrr} -2 & 3 & 10 & 5 & p \\ & & -6 & -8 & 6 \\ \hline & 3 & 4 & -3 & \boxed{p+6} \end{array}$$

But the remainder is zero.

$\therefore\ p + 6 = 0$

$\therefore\ p = -6$

Second factor $= 3x^2 + 4x - 3$

$\therefore\ 3x^3 + 10x^2 + 5x - 6 = (x+2)(3x^2 + 4x - 3)$]

Exercise 8.10

1. In the following examples, find whether the second polynomial is a factor of the first. In case it is a factor, find the other factor.

(1) $x^2 - 7x + 12$; $x - 3$ (2) $x^3 - 5x^2 + 7x - 3$; $x - 1$

(3) $x^3 + 3x^2 - 4x - 12$; $x - 2$ (4) $x^3 + 4x + 16$; $x + 2$

(5) $2y^3 - 9y^2 + 9y - 20$; $y - 4$ (6) $m^3 - 3m + 2$; $m + 2$

(7) $p^3 - 6p^2 - 12p - 5$; $p + 1$ (8) $y^3 + 2y^2 - 4y - 3$; $y + 3$

(9) $x^4 - 5x^3 + 6x^2 - x - 1$; $x - 1$

(10) $x^4 + x^3 - 5x^2 - 4x + 1$; $x + 1$

2. Find p, if $(x - 1)$ is a factor of $x^3 + 5x^2 + 4x - p$

3. Find p, if $(x - 2)$ is a factor $x^2 - px + 6$

4. Find a, if $(x + 3)$ is a factor of $x^2 + ax + 12$

5. If $y^2 + (a + 3)y - 15$ is divided by $y - 1$ the remainder is 5. find a.

6. Find a and b, if $(x - 1)$ and $(x - 2)$ are factors of $x^3 - ax + b$.

7. Find p and q if $(x - 1)$ and $(x + 1)$ are factors of $5x^3 + px^2 + qx + 7$.

8. Find a and b, if $(x - 1)$ and $(x - 3)$ are factors of $x^3 + ax^2 + bx + 9$.

9. $(x + 1)$ is a factor of $ax^4 + 5x^2 + 7x + 1$. Find a and the other factor.

10. $(x - 2)$ is a factor of $4x^4 - 5x^3 + bx^2 + 16$. Find b and the other factor.

§ 8.11 Tests of Divisibility :

We have learnt that $x - a$ is a factor of the polynomial P if

$$P\Big|_{x = a} = 0.$$

Determine whether $(x - 1)$ is a factor of the polynomial $P = ax^4 + bx^3 + cx^2 + dx + e$

$$P\Big|_{x = 1} = a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e$$

$$= a + b + c + d + e$$

$\therefore$ $(x-1)$ will be a factor of P, if $a+b+c+d+e=0$

Test 1 : for $(x-1)$

If the sum of the coefficients of a polynomial in x is zero, then $(x-1)$ is a factor of that polynomial.

Examples :-

1. Decide whether $(x-1)$ is a factor of the polynomial $4x^3 - 5x^2 + 6x - 5$ and if it is a factor, find the other factor.

$P = 4x^3 - 5x^2 + 6x - 5$

Sum of all coefficients in $P = 4 - 5 + 6 - 5 = 0$

$\therefore$ $(x-1)$ is a factor of P.

Let us find the other factor by synthetic division.

$$\begin{array}{r|rrrr} 1 & 4 & -5 & 6 & -5 \\ & & 4 & -1 & 5 \\ \hline & 4 & -1 & 5 & \boxed{0} \end{array}$$

$\therefore$ Second factor $= 4x^2 - x + 5$

Exercise 8.11

Decide whether $(x-1)$ is a factor of each of the following polynomials and if it is a factor, find the second factor.

(1) $x^3 - 5x^2 + 7x - 3$ (2) $2x^3 - 7x^2 - 5x + 10$

(3) $4x^3 - x^2 - 5x + 1$ (4) $3x^3 - 2x - 1$

(5) $x^3 - 2x^2 + 5x - 4$ (6) $2x^2 - 3x + 1$

(7) $x^3 - 27x + 26$ (8) $x^4 - 1$

(9) $9x^4 - 8x^3 + 10x^2 - 5x - 5$ (10) $5x^4 + 6x^3 - 11$

Determine whether $(x+1)$ is a factor of the polynomial

$P = ax^3 + bx^2 + cx + d$,

$(x+1)$ will be a factor of P, if $P\Big|_{x=-1} = 0$

$P = ax^3 + bx^2 + cx + d$

$$\therefore P\Big|_{x=-1} = a(-1)^3 + b(-1)^2 + c(-1) + d$$

$$= -a + b - c + d$$

$$= (b+d) - (a+c)$$

If $x+1$ is to be a factor of P, we should have $(b+d) - (a+c) = 0$ i.e. $b+d = a+c$.

Conversely this means that if $b+d = a+c$ then $x+1$ is a factor of P. a and c are coefficients of odd powers and b and d are coefficients of even powers in the polynomial P. From this we get following test for $(x+1)$ to be a factor of the polynomial P.

Test 2: for $(x+1)$

If for a polynomial P in x, the sum of the coefficients of even powers of x is equal to the sum of the coefficients of odd powers of x, then $(x+1)$ is a factor of the polynomial P,

If $(x+1)$ is a factor of P, the other factor can be obtained by synthetic division method.

Exercise 8.12

Decide whether $(x+1)$ is a factor of each of the following polynomial and if it is a factor, find the other factor.

(1) $x^2 + 5x + 4$ (2) $2x^3 - 5x^2 - 3x + 4$

(3) $x^3 - 5x^2 + 3x + 9$ (4) $2x^3 + 5x^2 + 4x + 1$

(5) $x^3 + 5x^2 + 6x + 2$ (6) $x^3 - 5x^2 - 7x - 1$

(7) $x^3 - 5x^2 + 6x + 12$ (8) $5x^4 + 3x^3 + x^2 + 4x + 2$

(9) $x^4 - 1$ (10) $ax^3 + bx^2 + (b+1)x + (a+1)$

§ 8.12 Factorization :

We know that $(x-a)$ is a factor of the polynomial P in x if value of the polynomial for $x = a$ is zero. The tests discussed above and synthetic division method enables us to find all first degree factors of the type $x-a$. you have studied other methods of factorization in eighth and ninth Standards. To split a given polynomial as far as possible and express it as a product of factors which cannot be factored further is called ' factorizing the given polynomial '.

Examples :

1. Factorize $x^3 + 2x^2 - 21x + 18$

Sum of all coefficients $= 1 + 2 - 21 + 18$

$= 0$

$\therefore$ $(x-1)$ is a factor of the given polynomial.

Let us decide the other factor.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -21 & 18 \\ & & 1 & 3 & -18 \\ \hline & 1 & 3 & -18 & \boxed{0} \end{array}$$

$\therefore$ the other factor is $x^2 + 3x - 18$

$\therefore$ $x^3 + 2x^2 - 21x + 18 = (x - 1)(x^2 + 3x - 18)$

$= (x - 1)(x + 6)(x - 3)$.

2. Factorize the poynomial $P = x^3 - 13x - 12$

In this example the sum of all coefficients in P is not zero. Hence $(x - 1)$ is not a factor of P.

Sum of the coefficients of even powers of $x = -12$

Sum of the coefficients of odd powers of $x = 1 - 13 = -12$

These sums are equal.

$\therefore$ $(x + 1)$ is a factor of the given polynomial.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$\therefore$ the other factor $= x^2 - x - 12$

Factorize and verify that $(x - 4)$ and $(x + 3)$ are factors of $x^2 - x - 12$.

$\therefore$ $x^3 - 13x - 12 = (x + 1)(x + 3)(x - 4)$

3. Factorize $P = y^3 - 4y^2 - 9y + 36$.

Verify that $y \pm 1$ are not factors of P.

In this example the constant term is 36. Factors of 36 are ± 1, ± 2, ± 3, ± 4, ± 6, ± 9, $+ 12$, ± 18 and ± 36.

$\therefore$ $y \pm 1$, $y \pm 2$, $y \pm 3$, $y \pm 4$, $y \pm 6$, $y \pm 9$, $y \pm 12$, $y \pm 18$ and $y \pm 36$ are possible factors of P. Out of these we have seen that $y \pm 1$ are not factors of P.

Let us find whether y-2 is a factor of P by synthetic division method.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -9 & 36 \\ & & 2 & -4 & -26 \\ \hline & 1 & -2 & -13 & \boxed{10} \end{array}$$

Remainder is not zero.

$\therefore$ $y-2$ is not a factor of P.

Similarly verify that $y+2$ is not a factor of P.

As ± 3 are factors of 36. Let us see whether $y-3$ is a factor of P by synthetic division method.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & -9 & 36 \\ & & 3 & -3 & -36 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

As the remainder is zero $y-3$ is a factor of P.

The other factor $= y^2 - y - 12$

Factorize $y^2 - y - 12$ and see that $(y-4)$ and $(y+3)$ are its factors,

$\therefore$ $y^3 - 4y^2 - 9y + 36 = (y-3)(y+3)(y-4)$

4. Factorize $P = x^3 - 4x^2 - 11x + 30$.

Verify that $x \pm 1$ are not factors of P.

Let us see whether $x-2$ is a factor by synthetic division method.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array}$$

Remainder is zero. $\therefore$ $x-2$ is a factor of P.

$\therefore$ $P = (x-2)(x^2 - 2x - 15)$

As $x \pm 1$ are not factors of the polynomial P, they cannot be factors of $x^2 - 2x - 15$.

As 3 is a factor of 15, let us see whether $x-3$ is a factor of $x^2 - 2x - 15$

$$\begin{array}{r|rrr} 3 & 1 & -2 & -15 \\ & & 3 & 3 \\ \hline & 1 & 1 & \boxed{-12} \end{array}$$

Remainder–$12 \neq 0$

$\therefore$ $x-3$ is not a factor of $x^2-2x-15$.

Now let us see whether $x+3$ is a factor of P.

$$\begin{array}{r|rrr} -3 & 1 & -2 & -15 \\ & & -3 & 15 \\ \hline & 1 & -5 & \boxed{0} \end{array}$$

Remainder $= 0$ $\therefore$ $(x+3)$ is a factor of $x^2-2x-15$. The other factor $= x-5$

$\therefore$ $x^3-4x^2-11x+30 = (x-2)(x+3)(x-5)$

5. Factorize $3y^3-7y^2-2y+8$.

$y-1$ is not a factor of the given polynomial because sum of coefficients is not zero.

$y+1$ is a factor of the given polynomial because $3-2=-7+8$.

Let us find the other factor by synthetic division method.

$$\begin{array}{r|rrrr} -1 & 3 & -7 & -2 & 8 \\ & & -3 & 10 & -8 \\ \hline & 3 & -10 & 8 & \boxed{0} \end{array}$$

$\therefore$ the other factor $= 3y^2-10y+8$.

Let us see whether $y-2$ is a factor of $3y^2-10y+8$, since 2 is a factor of 8.

$$\begin{array}{r|rrr} 2 & 3 & -10 & 8 \\ & & 6 & -8 \\ \hline & 3 & -4 & \boxed{0} \end{array}$$

Remainder $= 0$. $\therefore$ $y-2$ is one factor and the other factor is $3y-4$.

$\therefore$ $3x^3-7y^2-2y+8 = (y+1)(y-2)(3y-4)$

Exercise 8.13

Factoize the following polynomial,

(1) $x^3 + 2x^2 - x - 2$ (2) $y^3 - 7y + 6$ (3) $m^3 - 21m - 20$
(4) $x^3 - 5x^2 + 7x - 3$ (5) $y^3 - 4y^2 + y + 6$ (6) $y^3 + 3y^2 - y - 3$
(7) $x^3 + 6x^2 + 6x + 1$ (8) $x^3 + 6x^2 + 11x + 6$
(9) $m^3 + 10m^2 + 19m - 30$ (10) $p^3 - p^2 - 37p - 35$
(11) $y^3 + y^2 + 2y + 8$ (12) $x^3 + 9x^2 + 26x + 24$
(13) $x^3 - x^2 - x - 2$ (14) $x^3 - 37x + 84$
(15) $m^3 + m^2 - 36$ (16) $x^3 - 3x^2 - 54x + 112$
(17) $x^3 - 49x - 120$ (18) $y^3 - 79y + 210$
(19) $2x^3 - x^2 - 2x + 1$ (20) $x^4 - 13x^2 - 48$
(21) $2x^3 - 5x^2 - 22x - 15$ (22) $x^4 + 4x^3 + 6x^2 - 4x - 7$
(23) $x^4 - 5x^2 + 4$ (24) $6y^3 + 7y^2 - 7y - 6$

§ 8.13 G. C. D. and L. C. M. :

(1) Greatest Common Divisor : (G. C. D.)

You have learnt how to find greatest common divisor of two natural numbers. Now we wish to see how G. C. D. of two polynomials is found. Every monomial is a polynomial, hence let us first find the G. C. D. of two monomials.

1. Find the G. C. D, of $24x^2y^3z$ and $36x^3y^2$.

$$\begin{aligned} 24x^2y^3z &= 2 \times 2 \times 2 \times 3 \times x^2 \times y^3 \times z \\ &= 2^3 \times 3 \times x^2 \times y^3 \times z \\ 36x^3y^2 &= 2 \times 2 \times 3 \times 3 \times x^3 \times y^2 \\ &= 2^2 \times 3^2 \times x^3 \times y^2 \end{aligned}$$

In the above polynomials the maximum number of times 2 appears as a common factor is 2, the maximum number of times 3 appears as a common factor is 1 and maximum number of times x and y appear as common factor is 2 and 2 respectively.

$$\begin{aligned} \therefore \text{ the required G. C. D.} &= 2^2 \times 3 \times x^2 \times y^2 \\ &= 12x^2y^2 \end{aligned}$$

Note that z is **not a common factor** of these polynomials.

2. Find the G. C. D. of $(x-1)^3\,(x+2)\,(x-3)^3$ and $x\,(x-1)^2\,(x-3)^4$.

Here observe that maximum number of times $(x-1)$ appears as a common factor is 2 and maximum number of times $(x-3)$ appears as a common factor is 3. i. e. $(x-1)^2$ and $(x-3)^3$ are common factors. x and $(x+2)$ are not common factors.

$\therefore$ G. C. D. of given polynomials is $(x-1)^2\ (x-3)^3$.

3. Find the G. C. G. of $8(a+2b)(a-b)^3$ and $12(a+2b)(a+b)\ (a-b)$.

$8(a+2b)\ (a-b)^3 = 2^3(a+2b)\ (a-b)^3$

$12(a+2b)(a+b)(a-b) = 2^2 \times 3(a+2b)\ (a+b)(a-b)$

$\therefore$ the required G. C. D. $= 2^2(a+2b)\ (a-b)$

$= 4(a+2b)\ (a-b)$

4. Find the G. C. D. of $2(x^2-y^2)$, $x^2-2xy+y^2$ and $2x^4-2y^4$.

In this example we have to find the G. C. D. of three polynomials. Hence, first we factorize these three given polynomials.

$$2(x^2-y^2) = 2(x-y)\ (x+y)$$
$$x^2-2xy+y^2 = (x-y)^2$$
$$2(x^4-y^4) = 2(x^2-y^2)\ (x^2+y^2)$$
$$= 2(x-y)\ (x+y)\ (x^2+y^2)$$

In this example $(x-y)$ is the only factor which is common to all polynomials and the maximum number of times it appears as a common factor is 1.

$\therefore$ the G. C. D. of given polynomials $= (x-y)$

The above discussion indicates the procedure for finding G. C. D. of given polynomials. Factorize all the given polynomials. Express the polynomials in the form of product of powers of different factors. Decide the maximum power of each factor which is common to all polynomials. The product of all such maximum powers of common factors is the G. C. D. of the given polynomials.

Exercise 8.14

Find the G. C. D. of following polynomials,

(1) $24x$; $36x^2$ (2) $3x^2y$; $6x$

(3) x^2yz ; xz^2 (4) $(x-1)^2(x+2)$; $(x-1)(x+1)$

(5) $(y+1)^2\ (y-1)^2\ (y+2)$; $(y-1)^3$

(6) $(x-y)(x+y)(x+2y)$; $(x-y)^2(x+2y)$
(7) $15(x-1)(x+2)$; $3(x-1)^2$
(8) $4x^2-4y^2$; $2x+2y$
(9) $(x+3)^2$; $x^2+7x+12$
(10) x^2+5x+6 ; x^2+x-6
(11) $10(x^2-4y^2)$; $2(x-2y)^2$
(12) $(x-1)^3$; x^2-1 ; x^2+x-2

(2) Least Common Multiple. (L. C. M.) ;

1. Find the L. C. M. of $4x^3y^3$ and $6xy^2$

Let us find all factors of the given polynomials.

$4x^3y^3 = 2^2 \times x^3 \times y^3$: $6xy^2 = 2 \times 3 \times x \times y^2$

The factor 2 occurs twice in $4x^3y^3$ and once in $6xy^2$. Hence it must occur at least twice in their common multiple. Similarly the factors 3, x and y must occur in their common multiple at least once, thrice and thrice respectively.

$$\therefore \text{ L. C. M. of the given polynomials } = 2^2 \times 3 \times x^3 \times y^3 = 12x^3y^3$$

Thus, for finding L. C. M. of given polynomials, first factorize all the given polynomials. Then find the product of maximum power of each factor. This product is the least common multiple (L. C. M.) of the given polynomials.

2. Find the L.C.M. of $4(x-y)^2(x+2y)$ and $6(x-y)(x+y)(x+2y)$

Let us factorize the given polynomials.

$$4(x-y)^2(x+2y) = 2^2 \times (x-y)^2(x+2y)$$
$$6(x-y)(x+y)(x+2y) = 2 \times 3(x-y)(x+y)(x+2y)$$
$$\therefore \text{ the required L. C. M. } = 2^2 \times 3(x-y)^2(x+2y)(x+y)$$
$$= 12(x-y)^2(x+2y)(x+y)$$

3. Find the L. C. M. of x^3-1 and $x^3-6x^2+11x-6$.

Let us factorize the given polynomials.

x^3-1 can be factorized using the formula

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$x^3-1 = (x-1)(x^2+x+1)$

In the polynomial $x^3 - 6x^2 + 11x - 6$, sum of the coefficients is zero. Hence $x - 1$ is its factor.

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$\therefore$ other factor $= x^2 - 5x + 6 = (x-2)(x-3)$

$\therefore$ $x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

$\therefore$ required L. C. M. $= (x-1)(x-2)(x-3)(x^2+x+1)$

4. Find the G. C. D. and the L. C. M. of the polynomials $4x^2 - 36$, $2(x^2 + x - 12)$ and $6(x-3)(x^2-16)$.

Obtain the following factors of the given polynomials.

$$4x^2 - 36 = 2^2 (x-3)(x+3)$$
$$2(x^2 + x - 12) = 2(x-3)(x+4)$$
$$6(x-3)(x^2-16) = 2 \times 3 (x-3)(x-4)(x+4)$$

$\therefore$ G. C. D. $= 2(x-3)$

and L. C. M. $= 2^2 \times 3 (x-3)(x+3)(x+4)(x-4)$

Exercise 8.15

1. Find L. C. M, of the following polynomials.

(1) $12a^2$; $18a^3b$ (2) a^2b^4 ; a^3b^2

(3) $(x-a)^2(x+a)$; $(x-a)(x+a)^2$

(4) $(y-1)^2(y+3)$; $(y-1)(y-2)(y+3)$

(5) $(a-b)(a+2b)^2$; $(a-b)^2(a-2b)$

(6) $(m+2)(m-3)$; $(m+2)^2$

(7) $(x+2)(x-3)$; $(x-2)(x+3)$

(8) $4x^2(x-2)$; $6x(x+2)$

(9) $10(x^2-4y^2)$; $5(x-2y)^2$

(10) $x^2 - 5x + 6$; $x^2 - 9$

2. Find G. C. D. and L. C. M. of following polynomials.

(1) $x^2 + 5x + 6$; $x^2 - x - 12$ (2) $x^2 - y^2$; $x^3 - y^3$

(3) $y^2 - 16$; $y^2 - y - 12$ (4) $2x^3 - 2$; $x^2 - 2x + 1$

(5) $m^2 + 7m + 12$; $m + 4$ (6) $a^2 + 4a + 4$; $a^2 - 4$

(7) $x^3 + 27$; $2(x^2 - 3x + 9)$ (8) $a^2 - b^2$; $a^4 - b^4$

(9) $y^3 - 4y$; $y^2 - y - 2$ (10) $x^2 + 2x - 24$; $x^2 + x - 20$

(11) $x^3 - 6x^2 + 11x - 6$; $x^3 - 8$ (12) $x^2 - 1$; $x^3 - 4x^2 + 3x$

(13) $x^3 + 343$; $x^2 + 3x - 28$

(14) $y^3 + 6y^2 + 6y + 1$; $y^3 + 3y + 2$

(15) $x^3 - 3x + 2$; $x^3 - 7x + 6$

(16) $m^3 - 7m + 6$; $m^3 - 4m^2 + 5m - 2$

(17) $x^2 - 1$; $x^4 - 1$; $x^8 - 1$

(18) $2x^3 - 128$; $x^2 - 9x + 20$; $x^2 - 16$

(19) $7m^3 - 7$; $14m^2 - 14$; $21(m^2 - 2m + 1)$

(20) $x^2 - 4y^2$; $x^2 - xy - 2y^2$; $x^2 - 3xy + 2y^2$

(21) $6x^2 - 24$; $8x^2 - 8x - 48$; $2(x^2 + 5x + 6)$

(22) $a^4 + a^2 + 1$; $a^3 + 1$; $a^3 - 3a^2 + 3a - 2$

(23) $4x^2 - 9$; $2x^2 - 13x + 15$; $2x^2 + 7x - 15$

(24) $8y^2 - 200$; $2y^4 - 250y$; $4y^2 - 28y + 40$

(25) $2(x - 2)^2 (x + 3)$; $4x^2 - 36$; $8x^2 + 8x - 48$

Miscellaneous Exercise - 8

1. Simplify :

(1) $2(3x^2 - 5x + 7) - (4x^2 + 3x - 2) - 2(x^2 - 7x - 1)$

(2) $(a + b - c) + (b + c - a) + (c + a - b)$

(3) $(2m - n) + 3(m - 4n) - 2(2m + 5n)$

(4) $(p^2 + q^2 + 2pq) - 2(p^2 + q^2) + (p^2 + q^2 - 2pq)$

2. Multiply the polynomials in the following examples and state the degree of their products,

(1) $y^2 + y + 1$; $y - 1$ (2) $m^2 + m + 1$; $m^2 - m + 1$

(3) $a^2 - 3a + 4$; $2a - 3$ (4) $x^2 - 2x - 7$; $3x - 5$

3. Divide first polynomial by the second polynomial and express the result as dividend = divisor × quotient + remainder,

(1) $x^3 - 4x^2 + 5x - 3$; $x - 5$ (2) $y^3 - 3y^2 + 7$; $y^2 - 2$

(3) $4m^3 - 5m^2 + 7m$; $m + 3$ (4) $3x^3 - 5x^2 + 7x - 28$; $x - 2$

4. Find p, if $x - 1$ is a factor of the polynomial $x^3 - px^2 + 53x - 40$.

5. Factorize the following polynomials.

(1) $x^3 - 14x^2 + 63x - 90$ (2) $y^3 - 49y + 120$

(3) $m^3 + m^2 + 2m + 8$ (4) $a^4 - 8a$

(5) $x^3 + 3x^2 - 4x - 12$ (6) $x^3 + 5x^2 - x - 5$

(7) $a^3 + a^2 - 6a$ (8) $x^3 - 2x + 4$

(9) $x^4 - 1$ (10) $a^3 - 2a^2 - 5a + 6$

6. Find the G.C.D. and the L.C.M. and of the following polynomials.

(1) $3x^2(x-1)(x+2)$; $6x(x-1)^2(x+1)$

(2) $2x^3 - 16y^3$; $x^2 - 4xy + 4y^2$

(3) $x^3 + 6x^2 + 11x + 6$; $x^3 - 7x - 6$

(4) $4a^4 - 4b^4$; $6(a^2 - b^2)$

(5) $3x^2 - 147$; $x^8 + 343$

7. By Remainder Theorem, decide whether second polyomial is a factor of the first.

(1) $5y^3 - 12y + 16$; $y + 2$ (2) $x^3 - 3x^2 + 2x - 8$; $x + 4$

(3) $3x^3 + 6x^2 + 15x - 27$; $x + 3$ (4) $3x^4 - 2x^3 + 4x^2 + 12x + 3$; $x + 1$

8. Find c, if $x - 1$ is a factor of each of the following polynomials.

(1) $5x^3 - 6x + c$ (2) $x^3 - 8x^2 - cx + 5$

(3) $cx^3 - 5x^2 + 7x + 1$ (4) $x^3 + cx^2 - 5x + 7$

9. Divide first polynomial by the second using synthetic division method and express the result,

as dividend = divisor × quotient + remainder,

(1) $x^3 - 4x^2 + 5$; $x - 5$ (2) $y^3 - 5y^2 + 5y - 1$; $y - 1$

(3) $x^3 - 7x^2 + 5x - 4$; $x + 2$ (4) $2m^4 - 3m^2 + 5m - 15$; $m + 3$

10. If the value of the polynomial $2x^3 + ax^2 - 17x + b$ when $x = 2$ is 5 and when $x = -3$ is 45, find a and b.

● ● ● ●

Chapter 9

Rationl Algebraic Expressions

§ 9.1 Revision :

You have studied rational algebraic expressions in the ninth standard. $\frac{x+2}{x-1}$, $\frac{x}{y}$, $\frac{2x+3}{(x+4)(x-5)}$, $-2m$, 4, 0, $\frac{e^2+5e+6}{(e+2)(e-1)}$ are rational algebraic expressions.

If A and B are polynomials and B is a non-zero polynomial then the ratio $\frac{A}{B}$ is called a rational algebraic expression. Rational algebraic expression is obtained by dividing a polynomial by a non-zero polynomial, just as a rational number is obtained by dividing an integer by a non-zero integer.

If in the expression $\frac{2x+3}{(x+4)(x-5)}$, value of x is -4 or 5, then the denominator is zero. Hence the rational algebraic expression $\frac{2x+3}{(x+4)(x-5)}$ is meaningless for $x = -4$ or $x = 5$. For all other values of x it is meaningful.

Exercise 9.1

Complete the following table.

No.	Rational algebraic expression	Value of the variable	Expression meaningful/ meaningless	Value of the expression if it is meaningful
1	$\frac{x+2}{x-3}$	4	meaningful	$\frac{4+2}{4-3}=\frac{6}{1}=6$
2	$\frac{y-5}{y-4}$	5		
3	$\frac{x^2+5x}{x+2}$	-1		
4	$\frac{m^2-7m}{2m+1}$	$-\frac{1}{2}$		
5	$\frac{p}{(p+2)(p-3)}$	0		
6	$\frac{n+1}{n(n-2)}$	0		

The rational algebraic expression is not meaningful for those values of the variable, which make the value of the denominator zero. Henceforth in our discussion of rational algebraic expressions we shall assume that the polynomial in the denominator is non-zero.

§ 9.2

Rational number is reduced to its lowest term by using the property $\dfrac{a \times k}{b \times k} = \dfrac{a}{b}$,

Similarly the property $\dfrac{A \times K}{B \times K} = \dfrac{A}{B}$ is used for simplifying rational algebraic expressions. Note that the common factor in the numerator and the denominator of the rational algebraic expression is cancelled. Now let us study how this property is used for simplifying rational algebaic expressions.

Simplify.

(1) $\dfrac{m-3}{m^2-4m+3}$ (2) $\dfrac{a^3-b^3}{a^2-b^2}$ (3) $\dfrac{y^3+5y^2-2y-4}{y^2-3y+2}$

(1) $\dfrac{m-3}{m^2-4m+3} = \dfrac{(m-3)}{(m-1)(m-3)} = \dfrac{1}{m-1}$

(2) $\dfrac{a^3-b^3}{a^2-b^2} = \dfrac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)} = \dfrac{a^2+ab+b^2}{a+b}$

(3) $\dfrac{y^3+5y^2-2y-4}{y^2-3y+2} = \dfrac{(y-1)(y^2+6y+4)}{(y-1)(y-2)} = \dfrac{y^2+6y+4}{y-2}$

Exercise 9.2

Simplify.

(1) $\dfrac{5a^2b}{7a}$ (2) $\dfrac{x^2-4y^2}{(x-2y)(x-4y)}$ (3) $\dfrac{a^2+2ab+b^2}{a^2-b^2}$

(4) $\dfrac{x^3-1}{x^2+x+1}$ (5) $\dfrac{x^2+xy}{y^2+xy}$ (6) $\dfrac{x^2+5x+6}{x^2+x-2}$

(7) $\dfrac{a^3+b^3}{a^2+4ab+3b^2}$ (8) $\dfrac{5xy(x-y)}{10y^2(x-y)}$

(9) $\dfrac{x^3+3x^2-5x+1}{x^2-5x+4}$ (10) $\dfrac{y^3+3y^2-4y-6}{y^2-4y-5}$

§ 9.3

There is a close similarity of form between the rational algebraic expression and the rational number. Hence the properties of the rational algebraic expressions in respect of the four fundamental operations are also similar to those of the rational numbers.

Properties of rational numbers	**Properties of rational algebraic expressions**
$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$	$\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}.$

The numerator of the product is the product of the two numerators and the denominator of the product is the product of the two denominators.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$

to divide by is to multiply by reciprocal,

$\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	$\dfrac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}.$
$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$	$\frac{A}{B} \pm \frac{C}{B} = \frac{A \pm C}{B}$

For adding two rational algebraic expressions having the same denominator, the two numerators are added and the denominator is kept unchanged. Similar procedure is followed for subtraction.

Following property is used for addition or subtraction of two rational algebraic expressions having different denominators.

$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$	$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm BC}{BD}$
$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$	$\frac{A}{-B} = \frac{-A}{B} = -\frac{A}{B}$

Let us study how to use these properties in simplifying rational algebraic expressions.

Examples :

1. Simplify : $\dfrac{m^3+n^3}{m^2+4mn+3n^2} \div \dfrac{m^2+3n^2}{m+3n}$

$$\frac{m^3+n^3}{m^2+4mn+3n^2} \div \frac{m^2+3n^2}{m+3n}$$

$$= \frac{m^3+n^3}{m^2+4mn+3n^2} \times \frac{m+3n}{m^2+3n^2} \qquad \left(\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}\right)$$

$$= \frac{(m+n)(m^2-mn+n^2)}{(m+n)\ (m+3n)} \times \frac{(m+3n)}{m^2+3n^2} \qquad \text{(factorizing)}$$

$$= \frac{m^2-mn+n^2}{m+3n} \times \frac{m+3n}{m^2+3n^2} \qquad \left(\frac{A \times K}{B \times K} = \frac{A}{B}\right)$$

$$= \frac{(m^2-mn+n^2)(m+3n)}{(m+3n)(m^2+3n^2)} \qquad \left(\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}\right)$$

$$= \frac{m^2-mn+n^2}{m^2+3n^2} \qquad \left(\frac{A \times K}{B \times K} = \frac{A}{B}\right)$$

2. Simplify : $\dfrac{2x-3}{x-4} + \dfrac{x}{4-x}$

$$\frac{2x-3}{x-4} + \frac{x}{4-x}$$

$$= \frac{2x-3}{x-4} + \frac{x}{-(x-4)}$$

$$= \frac{2x-3}{x-4} - \frac{x}{x-4} \qquad \left(\frac{A}{-B} = -\frac{A}{B}\right)$$

$$= \frac{2x-3-x}{x-4} \qquad \left(\frac{A}{B} - \frac{C}{B} = \frac{A-C}{B}\right)$$

$$= \frac{x-3}{x-4}$$

3. Simplify : $\dfrac{y-2}{y^2+5y+6} - \dfrac{2y-3}{3(y^2-y-6)}$

In this example the denominators are not same. Let us see if they have a common factor by factorizing the denominators.

$$\frac{y-2}{y^2+5y+6}-\frac{2y-3}{3(y^2-y-6)}$$

$$=\frac{y-2}{(y+2)(y+3)}-\frac{2y-3}{3(y-3)(y+2)}$$

$$=\frac{(y-2)(y-3)\cdot 3}{(y+2)(y+3)(y-3)\cdot 3}-\frac{(2y-3)(y+3)}{3(y-3)(y+2)(y+3)}$$

(denominators are made same)

$$=\frac{3(y^2-5y+6)-(2y^2+3y-9)}{3(y+2)(y+3)(y-3)} \qquad \left(\frac{A}{B}-\frac{C}{B}=\frac{A-C}{B}\right)$$

$$=\frac{3y^2-15y+18-2y^2-3y+9}{3(y+2)(y+3)(y-3)}$$

$$=\frac{y^2-18y+27}{3(y+2)(y+3)(y-3)}$$

4. Simplify : $\dfrac{\dfrac{x+4}{x+5}}{\dfrac{x-1}{x+5}}$

$$\frac{\dfrac{x+4}{x+5}}{\dfrac{x-1}{x+5}}=\frac{x+4}{x+5}\div\frac{x-1}{x+5}$$

$$=\frac{x+4}{x+5}\times\frac{x+5}{x-1}$$

$$=\frac{(x+4)\times(x+5)}{(x+5)\times(x-1)}$$

$$=\frac{x+4}{x-1}$$

State the property used in each step.

5. Simplify : $\dfrac{\dfrac{3m-4}{m-3}+4}{\dfrac{4m-5}{m-3}+2}$

For simplifying such expression the numerator and the denominator should be simplified separately.

Numerator$= \dfrac{3m-4}{m-3}+4$

$= \dfrac{3m-4+4(m-3)}{m-3}$

$= \dfrac{3m-4+4m-12}{m-3}$

$= \dfrac{7m-16}{m-3}$

Denominator $= \dfrac{4m-5}{m-3}+2$

$= \dfrac{4m-5+2(m-3)}{m-3}$

$= \dfrac{4m-5+2m-6}{m-3}$

$= \dfrac{6m-11}{m-3}$

Given rational algebraic expression

= numerator. ÷ denominator

$= \dfrac{7m-16}{m-3} \div \dfrac{6m-11}{m-3}$

$= \dfrac{7m-16}{m-3} \times \dfrac{m-3}{6m-11}$

$= \dfrac{7m-16}{6m-11}$

6. If $y = \dfrac{2x-4}{4x-1}$, express $\dfrac{2y-4}{4y-1}$ in x and simplify.

$$\frac{2y-4}{4y-1} = \frac{2\left(\dfrac{2x-4}{4x-1}\right)-4}{4\left(\dfrac{2x-4}{4x-1}\right)-1}$$

Numerator$= 2\left(\dfrac{2x-4}{4x-1}\right)-4$

$= \dfrac{4x-8}{4x-1}-4$

$= \dfrac{4x-8-4(4x-1)}{4x-1}$

$= \dfrac{4x-8-16x+4}{4x-1}$

$= \dfrac{-12x-4}{4x-1}$

$= \dfrac{-4(3x+1)}{4x-1}$

Denominator $=4\left(\dfrac{2x-4}{4x-1}\right)-1$

$= \dfrac{8x-16}{4x-1}-1$

$= \dfrac{8x-16-1(4x-1)}{4x-1}$

$= \dfrac{8x-16-4x+1}{4x-x}$

$= \dfrac{4x-15}{4x-1}$

Given expression $= \frac{-4(3x+1)}{4x-1} \div \frac{4x-15}{4x-1}$

$$= \frac{-4(3x+1)}{4x-1} \times \frac{4x-1}{4x-15}$$

$$= \frac{-4(3x+1)}{4x-15}$$

$$= -\frac{4(3x+1)}{4x-15}$$

7. If $x = \frac{25}{53}$ and $y = \frac{28}{53}$, find the value of $\frac{x^3+y^3}{x-y} \div \left(x + \frac{y^2}{x-y}\right)$

Here the values of x and y are such that their cubes cannot be obtained easily.

Hence before substituting the values of the variables we shall simplify the given expression.

$$\frac{x^3+y^3}{x-y} \div \left(x + \frac{y^2}{x-y}\right)$$

$$= \frac{x^3+y^3}{x-y} \div \left(\frac{x^2-xy+y^2}{x-y}\right)$$

$$= \frac{(x+y)(x^2-xy+y^2)}{(x-y)} \times \frac{(x-y)}{(x^2-xy+y^2)}$$

$$= x+y$$

$$= \frac{25}{53} + \frac{28}{53} \text{ (Substituting values of } x \text{ and } y\text{)}$$

$$= \frac{53}{53} = 1$$

Exercise 9.3

1. Simplify :

(1) $\frac{m}{m-3} \times \frac{m+3}{m}$ (2) $\frac{x^2-y^2}{x+y} \times \frac{x}{x-y}$

(3) $\frac{4}{x+y} \div \frac{2}{x^2-y^2}$ (4) $\frac{m^2-n^2}{m^3+n^3} \div \frac{m+n}{m^2-mn+n^2}$

(5) $\frac{a+4}{a-2} \times \frac{a^2-5a+6}{a^2+a-12}$ (6) $\frac{a+4b}{a-2b} \times \frac{a^2-5ab+6b^2}{a^2+ab-12b^2}$

(7) $\frac{p^2-5p+4}{p-1} \div \frac{p-4}{p^2-1}$ (8) $\frac{p^2-5mp+4m^2}{p-m} \div \frac{p-4m}{p-m}$

(9) $\dfrac{3x^2+x-14}{2x^2-x-6} \div \dfrac{6x^2+5x-21}{4x^2-9}$ (10) $\dfrac{b^2-1}{b+1} \times \dfrac{b^3+2b^2}{b^2+b-2}$

(11) $\dfrac{a^2-16}{a^2+16a+64} \div \left(\dfrac{a^2-9a+18}{a^2-2a-48} \div \dfrac{a+8}{a+4}\right)$

(12) $\dfrac{x}{x^2+x-12} \div \left(\dfrac{x-5}{x-3} \div \dfrac{x^2-25}{x}\right)$

(13) $\dfrac{m}{m-n} - \dfrac{n}{m-n}$ (14) $\dfrac{a}{a+b} + \dfrac{b}{a+b}$

(15) $\dfrac{4q}{q^2-4} - \dfrac{q}{q+2}$ (16) $\dfrac{x^2}{x^2-y^2} - \dfrac{y}{y-x}$

(17) $\dfrac{m-8}{m-7} + \dfrac{m-4}{m-3}$ (18) $\dfrac{x^2-9}{x^2+5x+6} + \dfrac{x+3}{x+2}$

(19) $\dfrac{a-3}{a^2+a-12} - \dfrac{a+4}{a^2+9a+20}$

(20) $\dfrac{a-3b}{a^2+ab-12b^2} - \dfrac{a+4b}{a^2+9ab+20b^2}$

(21) $1-a+a^2-\dfrac{a^3}{1+a}$ (22) $\dfrac{1}{p+2} - \dfrac{2}{p^2-4}$

(23) $\left(\dfrac{p}{q}+\dfrac{r}{s}\right) \times qs - ps$

(24) $\left(\dfrac{m}{n}-\dfrac{n}{m}\right) \times mn \times \dfrac{m+n}{m^2-n^2}$

(25) $\dfrac{a^2-3a+2}{a^2+a-2} \div \dfrac{2a^2-5a+2}{2a^2+3a-2}$

(26) $\dfrac{a^2-3ab+2b^2}{a^2+ab-2b^2} \div \dfrac{2a^2-5ab+2b^2}{2a^2+3ab-2b^2}$

(27) $\dfrac{a}{a^2+a-12} \div \left(\dfrac{a-5}{a-3}-\dfrac{a+2}{a+4}\right)$

(28) $\dfrac{\dfrac{4x}{3}}{2+\dfrac{2x}{3}}$ (29) $\dfrac{a-\dfrac{1}{a}}{a+\dfrac{1}{a}}$ (30) $\dfrac{\dfrac{a}{b}-1}{\dfrac{a}{b}+1}$

(31) $\dfrac{1}{1+\dfrac{c}{a+b}} + \dfrac{1}{1+\dfrac{a}{b+c}} + \dfrac{1}{1+\dfrac{b}{c+a}}$

(32) $\dfrac{5x}{\dfrac{5x+15}{x}}$ (33) $\dfrac{3\left(\dfrac{2x-5}{x-3}\right)-5}{4\left(\dfrac{2x-5}{x-3}\right)-5}$

(34) $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}$ (35) $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

2. (1) If $x = \dfrac{2a-1}{4a+3}$, express $\dfrac{3x+2}{x-1}$ in a and simplify.

(2) If $y = \dfrac{2+x}{2-x}$, express $\dfrac{y+2}{y-2}$ in x and simplify.

(3) If $x = \dfrac{m}{2+m}$, express $\dfrac{3x}{x-1}$ in m and simplify.

3. Find the value of,

(1) $\dfrac{x^3-y^3}{x+y} \times \dfrac{1}{x+\dfrac{y^2}{x+y}}$ if $x = 100$ and $y = 99$

(2) $1+x+x^2+\dfrac{x^3}{1-x}$ if $x = 52$

(3) $\dfrac{x^2-y^2}{x^2+xy+y^2} \times \dfrac{x^3-y^3}{x^2-2xy+y^2}$ if $x = 0{\cdot}1$ and $y = 0{\cdot}9$

§ 9.4 Equations :

You have already studied how to solve equations in one variable. Now we shall study how to solve equations involving **rational** algebraic expressions.

Examples :-

1. Solve $\dfrac{1}{x+5}+\dfrac{1}{x+3} = \dfrac{1}{x+9}+\dfrac{1}{x-1}$

$$\frac{1}{x+5}+\frac{1}{x+3} = \frac{1}{x+9}+\frac{1}{x-1}$$

$$\therefore \frac{x+3+x+5}{(x+5)(x+3)} = \frac{x-1+x+9}{(x+9)(x-1)}$$

$$\therefore \frac{2x+8}{x^2+8x+15} = \frac{2x+8}{x^2+8x-9}$$

Either $2x+8=0$ or $2x+8=0$

If $2x+8 \neq 0$ then dividing both sides of the equation by $2x+8$ we get,

$$\frac{1}{x^2+8x+15} = \frac{1}{x^2+8x-9}$$

$\therefore\ x^2+8x+15 = x^2+8x-9$

$\therefore\ 15 = -9$

But this is absurd.

$\therefore\ 2x+8=0$ must hold.

$\therefore\ 2x=-8 \quad \therefore\ x=-4$

Verification :- Substituting $x=-4$ in the given equation

$$\frac{1}{-4+5} + \frac{1}{-4+3} \overset{?}{=} \frac{1}{-4+9} + \frac{1}{-4-1}$$

$$\frac{1}{1} + \frac{1}{-1} \overset{?}{=} \frac{1}{5} + \frac{1}{-5}$$

$$1 - 1 \overset{?}{=} \frac{1}{5} - \frac{1}{5}$$

$$0 \overset{\checkmark}{=} 0$$

$\therefore$ Solution set $= \{-4\}$

2. Solve the following equation.

$$\frac{1}{x-6} - \frac{1}{x-7} = \frac{1}{x-2} - \frac{1}{x-3}$$

$$\frac{1}{x-6} - \frac{1}{x-7} = \frac{1}{x-2} - \frac{1}{x-3}$$

$$\therefore\ \frac{x-7-(x-6)}{(x-6)(x-7)} = \frac{x-3-(x-2)}{(x-2)(x-3)}$$

$$\therefore\ \frac{x-7-x+6}{x^2-13x+42} = \frac{x-3-x+2}{x^2-5x+6}$$

$$\therefore\ \frac{-1}{x^2-13x+42} = \frac{-1}{x^2-5x+6}$$

$\therefore\ x^2-13x+42 = x^2-5x+6$

$\therefore\ -13x+5x = 6-42$

$$\therefore \quad -8x = -36$$

$$\therefore \quad x = \frac{-36}{-8} = \frac{9}{2}$$

Substitute $x = \frac{9}{2}$ in the given equation and verify.

$\therefore$ Solution set $= \{ \frac{9}{2} \}$

3. Solve :- $\dfrac{1}{4m+5} + \dfrac{1}{4m+3} = \dfrac{1}{4m+9} + \dfrac{1}{4m-1}$

In the above equation if we put x for $4m$ we get the following equation,

$$\frac{1}{x+5} + \frac{1}{x+3} = \frac{1}{x+9} + \frac{1}{x-1}$$

In example 1 we have seen that this equation when solved gives $x = -4$

$\therefore$ $4m = -4$ $\quad\therefore$ $m = -1$

$\therefore$ Solution set of the given equation in m is $= \{-1\}$

4. Solve :- $\dfrac{x-8}{x-7} - \dfrac{x-6}{x-5} = \dfrac{x-5}{x-4} - \dfrac{x-3}{x-2}$

$$\frac{x-8}{x-7} - \frac{x-6}{x-5} = \frac{x-5}{x-4} - \frac{x-3}{x-2}$$

$$\therefore \frac{(x-7)-1}{x-7} - \frac{(x-5)-1}{x-5} = \frac{(x-4)-1}{x-4} - \frac{(x-2)-1}{x-2}$$

$$\therefore 1 - \frac{1}{x-7} - 1 + \frac{1}{x-5} = 1 - \frac{1}{x-4} - 1 + \frac{1}{x-2}$$

$$\therefore \frac{1}{x-5} - \frac{1}{x-7} = \frac{1}{x-2} - \frac{1}{x-4}$$

$$\therefore \frac{x-7-(x-5)}{(x-5)(x-7)} = \frac{x-4-(x-2)}{(x-2)(x-4)}$$

$$\therefore \frac{x-7-x+5}{x^2-12x+35} = \frac{x-4-x+2}{x^2-6x+8}$$

$$\therefore \frac{-2}{x^2-12x+35} = \frac{-2}{x^2-6x+8}$$

$$\therefore \quad x^2-6x+8 = x^2-12x+35$$

$$\therefore \quad 12x-6x = 35-8$$

$\therefore \quad 6x = 27$

$\therefore \quad x = \frac{9}{2}$

$\therefore$ Solution set $= \{ \frac{9}{2} \}$

Exercise 9.4

Solve :-

(1) $\frac{1}{x-4} + \frac{1}{x-8} = \frac{1}{x-5} + \frac{1}{x-7}$

(2) $\frac{1}{3m-4} + \frac{1}{3m-8} = \frac{1}{3m-5} + \frac{1}{3m-7}$

(3) $\frac{1}{x-7} + \frac{1}{x-4} = \frac{1}{x-10} + \frac{1}{x-1}$

(4) $\frac{1}{x+1} - \frac{1}{x-3} = \frac{1}{x-7} - \frac{1}{x-11}$

(5) $\frac{1}{y+6} + \frac{1}{y+3} = \frac{1}{y+4} + \frac{1}{y+5}$

(6) $\frac{1}{4m+6} + \frac{1}{4m+3} = \frac{1}{4m+4} + \frac{1}{4m+5}$

(7) $\frac{1}{y-7} - \frac{1}{y+1} = \frac{1}{y-5} - \frac{1}{y+3}$

(8) $\frac{1}{y+17} + \frac{1}{y+3} - \frac{1}{y+5} = \frac{1}{y+15}$

(9) $\frac{1}{x-11} - \frac{1}{x-17} = \frac{1}{x+1} - \frac{1}{x-5}$

(10) $\frac{x-8}{x-7} + \frac{x-5}{x-4} = \frac{x-11}{x-10} + \frac{x-2}{x-1}$

(11) $\frac{x-3}{x-5} + \frac{x-2}{x-4} = \frac{x-4}{x-6} + \frac{x-1}{x-3}$

(12) $\frac{x+16}{x+17} + \frac{x+2}{x+3} = \frac{x+14}{x+15} + \frac{x+4}{x+5}$

(13) $\frac{y-6}{y-7} + \frac{y-10}{y-11} = \frac{y-7}{y-8} + \frac{y-9}{y-10}$

(14) $\frac{x-3}{x-5} - \frac{x-6}{x-8} = \frac{x+1}{x-1} - \frac{x-2}{x-4}$

(15) $\dfrac{x+2}{x}+\dfrac{x+5}{x+3} = \dfrac{x+1}{x-1}+\dfrac{x+6}{x+4}$

(16) $\dfrac{x+3}{x+2}+\dfrac{x+9}{x+8} = \dfrac{x+7}{x+6}+\dfrac{x+5}{x+4}$

(17) $\dfrac{1}{y-2}+\dfrac{1}{y-6} = \dfrac{1}{y-1}+\dfrac{1}{y-7}$

(18) $\dfrac{1}{x+4} - \dfrac{1}{x} = \dfrac{1}{x+3}-\dfrac{1}{x-1}$

(19) $\dfrac{1}{2y+5}+\dfrac{1}{2y+7} = \dfrac{1}{2y+10}+\dfrac{1}{2y+2}$

(20) $\dfrac{4x-11}{x-3}+\dfrac{4x-27}{x-7} = \dfrac{4x-23}{x-6}+\dfrac{4x-15}{x-4}$

Miscellaneous Exercise - 9

1. Simplify,

(1) $\dfrac{15x^3y^2}{3xy}$ (2) $\dfrac{m^2-4mn+4n^2}{(m-2n)(m+3n)}$ (3) $\dfrac{a^3-b^3}{a^4-b^4}$

(4) $\dfrac{x^2-5x+6}{x^2-4x+3}$ (5) $\dfrac{a^2-a-6}{a^2+6a+8}$ (6) $\dfrac{x^3+5x^2+7x+3}{x^4+x}$

2. Simplify.

(1) $\dfrac{a^2+2a-15}{a^2-25}\times\dfrac{a^2-8a+15}{a^2-9}$ (2) $\dfrac{x^2+4x-21}{x^2-9x+14}\div\dfrac{x-3}{x^2+x-6}$

(3) $\dfrac{x^3+1}{x^4+x^2+1}\times\dfrac{x^2+x+1}{x^3+4x^2-x-4}$ (4) $\dfrac{2x^3+5x^2-7x}{x^2-1}\div\dfrac{2x^2+7x}{x+1}$

(5) $\dfrac{3x}{x-4}+\dfrac{12}{4-x}$ (6) $\dfrac{m+2}{2(m^2-4m+3)}-\dfrac{1}{3m-3}$

(7) $\dfrac{\dfrac{m+1}{m}-1}{\dfrac{m-1}{m}+1}$ (8) $\dfrac{3\left(\dfrac{x-1}{x+2}\right)-2}{4\left(\dfrac{x-1}{x+2}\right)-3}$

(9) $\dfrac{p-\dfrac{1}{p}}{p+1}$ (10) $\dfrac{(x+h)^3-x^3}{h}$

3. Solve,

(1) $\dfrac{1}{x+3}+\dfrac{1}{x+4}=\dfrac{1}{x+2}+\dfrac{1}{x+5}$

(2) $\dfrac{1}{y-5}+\dfrac{1}{y-3}=\dfrac{1}{y-6}+\dfrac{1}{y-2}$

(3) $\dfrac{1}{3m-7}+\dfrac{1}{3m-4}=\dfrac{1}{3m-10}+\dfrac{1}{3m-1}$

(4) $\dfrac{1}{x+4}+\dfrac{1}{x+7}=\dfrac{1}{x+5}+\dfrac{1}{x+6}$

(5) $\dfrac{1}{y-4}-\dfrac{1}{y-3}=\dfrac{1}{y-6}-\dfrac{1}{y-5}$

(6) $\dfrac{1}{x+3}+\dfrac{1}{x-5}=\dfrac{1}{x-7}+\dfrac{1}{x+5}$

(7) $\dfrac{y-5}{y-4}+\dfrac{y-8}{y-7}=\dfrac{y-10}{y-9}+\dfrac{y-3}{y-2}$

(8) $\dfrac{x-4}{x-5}-\dfrac{x+4}{x+3}=\dfrac{x+2}{x+1}-\dfrac{x+10}{x+9}$

(9) $\dfrac{x+5}{x+6}+\dfrac{x+1}{x+2}=\dfrac{x+2}{x+3}+\dfrac{x+4}{x+5}$

(10) $\dfrac{1}{3p+1}-\dfrac{1}{3p+7}=\dfrac{1}{3p-15}-\dfrac{1}{3(p-3)}$

Chapter 10
Variation

§ 10.1

When we wish to purchase a thing sometimes it happens that we do not know its actual price. But we definitely know that the price will vary in proportion to the number of things purchased varies. If the number of things to be purchased is increased by a certain ratio, the price will increase in the same ratio. If the number of things to be purchased is decreased by some ratio, the price will also decrease in the same ratio. We say that prices of things purchased is in proportion to the number of things purchased.

Below is shown a table of prices corresponding to the number of notebooks. We use the letter x to show the number of notebooks and the letter y to show their total price.

x	2	3	6	8	15	35	85
y Rs.	4	6	12	16	–	–	–

How will you determine the price of 15 notebooks from the information supplied in the above table ?

In the second column of the above table the price of 2 notebooks is shown as Rs. 4. So the price of one notebook will be $\frac{4}{2} = 2$ rupees. Therefore, the price of 15 notebooks will be Rs. 30 In this way we can also find the price of 35 notebooks or 85 notebooks. The price of one notebook is obtained from the information contained in the second column. This price could have been determined from the information supplied in 3rd, 4th or 5th column. To find the price of a notebook we divide the total price by the number of notebooks corresponding to that price. But, $\frac{6}{3} = \frac{12}{6} = \frac{16}{8} = 2$ rupees. Each time the price per notebook comes out to be the same. This is but natural.

In the above table the numbers written against x and y are said to be values of x and y respectively. From column to column these values seem to vary. For this reason ; x and y are said to be variables. From the above table we obsesre that variation in values of y depends on the variation in values of x. (The word variation is sometimes used for the difference between two values of a variable. The word variation used here is not meant to carry this meaning.) The property which allows x and y to acquire changes in their values is known as **variation.**

In the example above both x and y vary but while varying they vary in such a way that $\frac{y}{x}$ is a constant.

If variables x and y are such that for any two correspanding values of these variables, we have the value of $\frac{y}{x}$ as constant, we say that y varies directly as x or y is proportional to x, or variation in y is the same as variation in x. In symbols, this is exprerssed as

$$y \propto x$$

If the constant value of $\frac{y}{x}$ is denoted by k then, k is called the constant of variation.

So, when $y \propto x$, we have $y = kx$, for some k, and when for some k, $y = kx$, we have $y \propto x$. It is obvious that k here is the constant of variation.

Suppose, that $y \propto x$ and that corresponding to values x_1, x_2, x_3,...... of x, the values of y are respectively y_1, y_2, y_3, ... Since $y \propto x$ we have,

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \ldots\ldots$$

We have seen what is meant by saying that four numbers a, b, c, d are in proportion. We will now extend this idea to many more numbers.

Suppose, that x_1, x_2, x_3....... are the values of x and that the corresponding values of y are y_1, y_2, y_3....... To say that numbers $x_1, x_2\ x_3$ ...... are in proportion to the numbers y_1, y_2, y_3, ... is the same as saying,

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \ldots\ldots\ \text{holds.}$$

The above discussion brings out the similarity between the notions of ' being in direct variation ' and ' being is proportion. '

The sentences, ' y varies directly as x ' and ' y varies in the same proportion as x ' convey the same meaning.

§ 10.2 Solved Exampls :

1 If $x \propto y$ and $x = 15$ when $y = 5$, find, (i) y when $x = 9$ (ii) x when $y = 8$.

$$x \propto y$$

$$\therefore \frac{x}{y} = k \text{ ...... (} k \text{ is a constant)}$$

When $x = 15$. $y = 5$

$$\therefore \frac{x}{y} = \frac{15}{5} = 3 = k.$$

$$\therefore \frac{x}{y} = 3 \text{ or } x = 3y.$$

Now, (i) substitute $x = 9$ in $x = 3y$

$\therefore 9 = 3y \quad \therefore y = 3$

(ii) Substitute $y = 8$ in $x = 3y$

$\therefore x = 3 \times 8 = 24$.

2. Monthly expenditure of a person varies directly as his monthly salary drawn. When his monthly salary was Rs. 800/- his monthly expenditure was Rs. 640. find,

(1) his monthly expenditure when his monthly salary was Rs. 1300/-

(2) his present salary, if his present monthly expenditure is Rs. 1200/-

Let us denote the monthly expenditure in Rs. of that person by x and his monthly salary in Rs. by y.

$$x \propto y$$

$$\therefore \frac{x}{y} = k \text{ .... (} k \text{ is a constant)}$$

Now, when $y = 800$, $x = 640$

$\therefore \frac{640}{800} = \frac{4}{5} = k.$

$$\therefore \frac{x}{y} = \frac{4}{5} \text{ or } x = \frac{4}{5} y.$$

(i) Substituting $y = 1300$ in $x = \frac{4}{5}y$,

we get, $x = \frac{4}{5} \times 1300 = 1040$

$\therefore$ Monthly expenditure is Rs. 1040.

(ii) Substituting, $x = 1200$ in $x = \frac{4}{5}y$

we get, $1200 = \frac{4}{5}y$

$\therefore \quad 1200 \times \frac{5}{4} = y$

$\therefore \quad 1500 = y$

$\therefore$ Monthly salary is Rs. 1500.

Exercise 10.1

1. Assuming $y \propto x$, find the constant of variation and write down the equation that expresses the relation of y to x.

   (i) $x = 5$, $y = 3$ (ii) $x = 7$, $y = 22$ (iii) $x = 3$, $y = 4$

   (iv) $x = 8$, $y = 13$ (v) $x = 1{\cdot}2$, $y = {\cdot}5$ (vi) $x = 2$, $y = 5.2$

2. If $y \propto x$ and $x = 20$ when $y = 24$, find

   (i) x when $y = 60$ (ii) y when $x = 25$.

3. If $s \propto t$ and $s = 50$ when $t = 2$, find

   (i) s when $t = 5$ (ii) t when $s = 200$

4. The mass of a metal ball varies directly as its volume. If its mass is 12 gms. when its volume is 8 cm^3, find the volume when the mass is 30gms.

5. The price of wheat varies directly as its weight. When its weight is 15 kg its price is Rs. 33. Find the price when its weight is one quintal.

§ 10.3 Inverse Variation :

A vehicle proceeds from town *A* to town *B*. The faster the speed of the vehicle lesser will be the time taken by it to reach town *B*. In other words more speed will mean less time, and conversely less speed will mean more time to cover the same distance.

In the table below values of speed (v) in km. per hour and time (t) in hours taken by the same vehicle at that speed to reach town *B* is given.

v	5	10	20	25
t	20	10	5	–

From the above table can you decide in what time did that vehicle reach town *B*, when its speed was 25 km./hr. ?

To be able to answer this question we must know the distance between town *A* and town *B*. From the information supplied in the second column time taken by the vehicle to reach town *B*, when the speed is 5 km./hr., is 20 hours. So, the distance between the two towns is $20 \times 5 = 100$ km. If this distance is calculated from the information supplied in the third or fourth column, it comes out to be the same i.e. 100 km. In this table, the product of the numbers in any column, is constant; being 100 in each case. Using this fact we can now easily find out the time taken by the vehicle to reach the town *B* at speed 25 km./hr. The answer is 4 hours.

In this example, the values of the variables v and t change, but they do so in such a way that their product remains constant. In such situations we say v varies inversely as t, or that there is inverse proportion in v and t.

In general, if values of the variables x and y depend on each other in such a way that the product of their corresponding values is constant, then we say that there is inverse variation in x and y or that x varies inversely as y.

Now, suppose that variables x and y are in inverse proportion. Let $x_1, x_2, x_3 \ldots\ldots$ be values of the variable x, when the variable y assumes corresponding values $y_1, y_2, y_3, \ldots\ldots$

Because, x varies inversely as y, we have

$$x_1y_1 = x_2y_2 = x_3y_3 = \ldots\ldots\ldots = \text{k (a constant)}$$

We may write this as,

$$\frac{x_1}{\frac{1}{y_1}} = \frac{x_2}{\frac{1}{y_2}} = \frac{x_3}{\frac{1}{y_3}} = \ldots\ldots\ldots = k$$

Using the symbol $\propto$, we may now say

$$x \propto \frac{1}{y}$$

Thus the sentence x varies inversely to y can be expressed in symbols as

$$x \propto \frac{1}{y}$$

In short, when the ratio of the corresponding values of the two variables is constant, we say that variation in these variables is direct, or that there is a direct variation in these variables. On the other hand when the product of the corresponding values of the variables is constant we say that there is inverse variation in these variables or one variable varies inversely with the other.

§ 10.4 Graphs of direct and inverse variation :

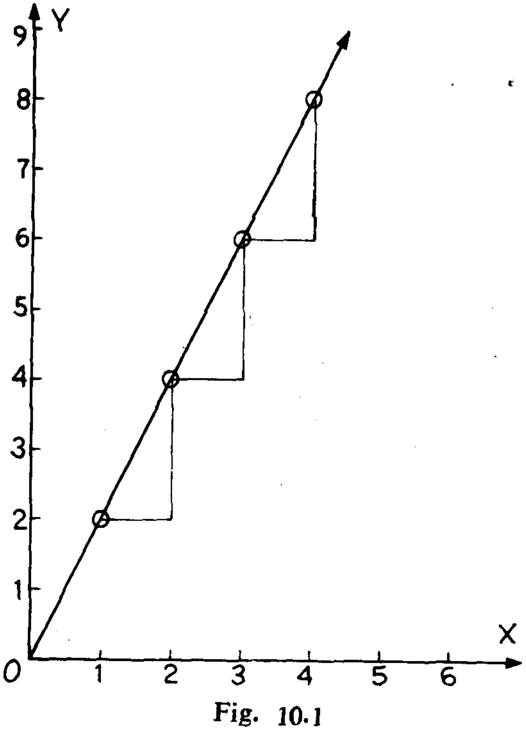


Fig. 10.1

Fig. 10.1 shows the graph of direct variation. When $y \propto x$, we have $\frac{y}{x} = k$ or $y = kx$ (k is a constant.). When $k = 2$ we have $\frac{y}{x} = 2$ or $y = 2x$ the graph of $y = 2x$ is a line passing through the origin. In the graph only half line is shown.

Corresponding to non-negative values of x and y, from the graph it is clear that changes in values of y are directly proportional to changes in values of x.

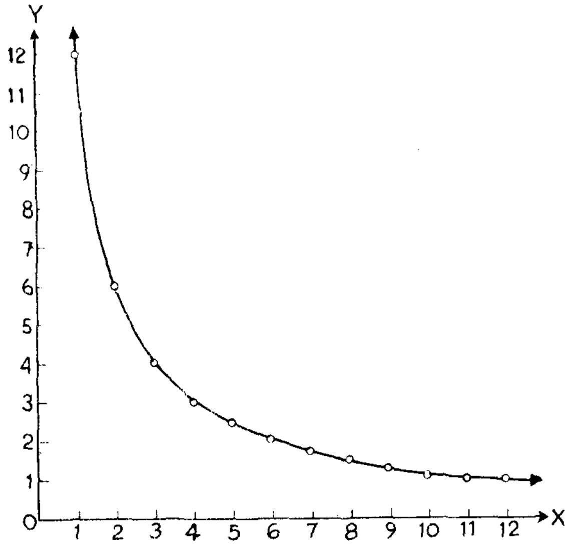


Fig. 10.2

Fig. 10.2 shows the graph of inverse variation, $y \propto \frac{1}{x}$ i. e. $xy = k$ (k is a constant). For $k = 12$, $xy = 12$. The graph in 10.2 shows the graph of $xy = 12$. Graph is drawn taking into consideration non-negative values of x and y. The graph is the portion of a figure known as hyperbola.

Now, read the graph in Fig. 10.2 and answer the following,

(1) find y when $x = 3$

(2) find x when $y = 1$

(3) find y when $x = 5$

(4) find x when $y = 8$

(5) Do the values you have obtained satisfy $xy = 12$?

(6) Will the graph of hyperbola $xy = 12$, intersect the axes X or Y ?

§ 10.5 Solved examples :

1. If $x \propto \frac{1}{y}$ and $x = 6$ when $y = 4$, find (i) y when $x = 8$

(ii) x when $y = 1.5$

$$x \propto \frac{1}{y}$$

$\therefore\ xy = k$ ...... (k is a constant)

when $x = 6$, $y = 4$

$\therefore\ 6 \times 4 = k$

$\therefore\ 24 = k$

$\therefore\ xy = 24$ or $y = \frac{24}{x}$

(i) Substitute $x = 8$ in $xy = 24$

$\therefore\ 8y = 24$

$\therefore\ y = 3$

(ii) Substitute $y = 1.5$ in $xy = 24$

$\therefore\ 1.5\,x = 24$

$$\therefore\ x = \frac{24}{1.5}$$

$$= \frac{24 \times 10}{15}$$

$$= 16$$

2. We know that the number of days required to complete a job varies inveresely with the number of workers appointed to do the job.If 6 workers complete a job in 10 days, how many workers will be required to complete the same job in 4 days ?

Let x denote the number of days and y denote the number of workers.

We have, $x \propto \frac{1}{y}$,

$\therefore\ xy = k$ ...... (k is a constant)

Since, 6 workers complete the job in 10 days, we have, $y = 6$ when $x = 10$.

After substituting in $xy = k$, we have,

$10 \times 6 = k$

$\therefore\ 60 = k$

$\therefore\ xy = 60$

Since, the work is to be completed is 4 days. We substitute 4 for y in $xy = 60$,

we have, then, $x \times 4 = 60$

$\therefore x = 15$

$\therefore$ 15 workers should be appinted.

Exercise 10.2

1. Assuming $y \propto \frac{1}{x}$, find the value of constant k in each example given below. In each case write the equations showing how x is related to y.
   (i) $x = 30$, $y = 5$ (ii) $x = 1.5$, $y = 10$ (iii) $x = \frac{1}{4}$, $y = 32$
   (iv) $x = 2.4$, $y = 1.2$ (v) $x = 10^2$, $y = 10$ (vi) $x = 45$, $y = 4$

2. If $y \propto \frac{1}{x}$ and $x = 4$ when $y = 5$, find (i) y when $x = 2.5$ (ii) x when $y = 50$

3. If $y \propto \frac{1}{x}$ and $y = 16$ when $x = 4$ find (i) y when $x = 12$ (ii) x when $y = 256$.

4. The time taken by a vehicle to reach a town B from a town A, varies inversely as its speed. A vehicle running at a speed of 40km./hr. takes 5 hours to complete the journey. What should be its speed to complete the journey in 4 hours ?

5. The time taken for filling a tank by a tap varies inversely as the speed with which water flows in to the tank. If it takes 5 hours for the tank to be completely filled when water flows in with a speed of 80 liters per hour, find the time required to fill the tank completely if water flows in with speed of 100 litres per hour.

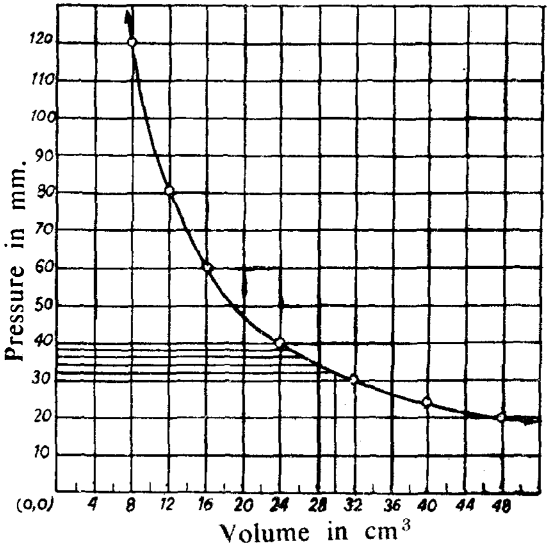


Fig. 10.3.

6. The relation between the pressure and the volume of an enclosed gas at a particular temperature is shown by the graph in fig. 10.3 Read the graph and answer the following :

(1) Find the pressure in m.m. of an enclosed gas when its volume is 24 cm^3.

(2) Find the volume in cm^3 of the enclosed gas when its pressure is 32 mm.

(3) What type of variation is between the pressure and the volume of the enclosed gas ? What is the constant of variation ?

(4) Can the numbers showing pressure and volume of the enclosed gas be equal ? If so, what is that number ?

§ 10.6 Problems in variation connected with standard formulae :

1. The formula connected with the circumference of a circle with its diameter is $C = \pi D$. where C is the circumference and D is the diameter. Hence C and D may be treated as variables. π is a constant. C and D vary in such a way that $\frac{C}{D} = \pi$

$$\therefore \quad C \propto D$$

Hence, we say that the circumference of a circle varies directly as its diameter. We also say that circumference of a circle is directly proportional to its diameter.

2. The formula for area of a circle is

$$A = \pi R^2$$

Here, A denotes area and R denotes the radius of the circle. We may treat A and R as variables. π is a constant. A and R vary in such a way that $\frac{A}{R_2} = \pi$; so that here

$$A \propto R^2$$

We, therefore, say that area of a circle varies directly as the square of its radius.

Note that it would be wrong to say that area varies directly as its radius, for in that case we would have $A = KR$..

3. Formula for the volume of sphere is $V = \frac{4}{3}\pi R^3$. Here V stands for volume and R for its radius. The term $\frac{4}{3}\pi$ is of course a constant. V and R vary in such a way that $\frac{V}{R^3} = \frac{4}{3}\pi$ (a constant)

We therefore have $V \propto R^3$ (and not $V \propto R$) volume of a sphere varies directly as the cube of its radius.

4. Formula for the area of a rectangle is $A = L \times B$. Here A stands for area. L for its length and B for its breadth. If we consider rectangles of the same area we must treat A as a constant. In this case variables L and B vary in such a way that value of $L \times B$ is a constant. So we can say that L varies inversely as B. $\left(L \propto \frac{1}{B}\right)$

5. Formula of the area of a triangle is $A = \frac{1}{2} B \times H$. where A stands for area, B for base and H for its height w. r. t. the base.

As above if we consider all triangles having the same area, A becomes a constant and variables B and H vary in such a way that the value of $B \times H$ is constant.

$$\therefore B \propto \frac{1}{H}$$

§ 10.7 Solved Examples :

1. $y \propto x^2$. when $x = 4$, $y = 2$. Find (i) x when $y = 18$ (ii) y when $x = 6$.

$$y \propto x^2$$

$$\therefore \quad \frac{y}{x^2} = k \qquad (k \text{ is a constant})$$

Since, when $x = 4$, $y = 2$, we have

$$\frac{2}{16} = k$$

$$\therefore \quad \frac{1}{8} = k.$$

$$\therefore \quad \frac{y}{x^2} = \frac{1}{8} \quad \text{or} \quad y = \frac{1}{8}x^2$$

(i) We substitute $y = 18$ in $y = \frac{1}{8}x^2$

$$\therefore \quad 18 = \frac{1}{8}x^2$$

$$\therefore \quad 144 = x^2$$

$$\therefore \quad +12 = x$$

(ii) Substituting the value $x = 6$ in $y = \frac{1}{8}x^2$

we get,

$$y = \frac{1}{8} \times 36$$

$$= \frac{9}{2}$$

2. $y \propto \frac{1}{\sqrt{x}}$ and when $x = 9$, $y = 4$; find x when $y = 3$.

$$y \propto \frac{1}{\sqrt{x}}$$

$$\therefore \quad y \times \sqrt{x} = k \ldots\ldots\ldots (k \text{ is a constant})$$

when $x = 9$, $y = 4$

$$\therefore \quad 4 \times \sqrt{9} = k$$

$$\therefore \quad 4 \times 3 = k$$

$$\therefore \quad k = 12$$ and we get.

$$y \times \sqrt{x} = 12$$

Substituting $y = 3$ in $y \times \sqrt{x} = 12$ we get

$$3 \times \sqrt{x} = 12$$

$$\therefore \quad \sqrt{x} = 4$$

$$\therefore \quad x = 16.$$

3. Volume of a sphere varies directly as the cube of its radius. When the radius of the sphere is $\frac{7}{2}$ cm. its volume is $\frac{539}{3}$ cm ^{3} What is the volume of a sphere where radius is $\frac{3}{2}$ cm. ?

Let V donote the volume of the sphere and R its radius. Then, we have,

$$V \propto R^3$$

$$\therefore \quad V = KR^3 \qquad (K \text{ is a constant})$$

$$\text{when} \quad R = \frac{7}{2}, \quad V = \frac{539}{8}$$

$$\therefore \quad \frac{539}{3} = K \times \left(\frac{7}{2}\right)^3$$

$$\therefore \quad \frac{539}{3} = K \times \frac{7 \times 7 \times 7}{2 \times 2 \times 2}$$

$$\therefore \quad K = \frac{539}{3} \times \frac{2 \times 2 \times 2}{7 \times 7 \times 7} = \frac{88}{21}$$

$$\therefore \quad \frac{V}{R^3} = \frac{88}{21}$$

$$\therefore \quad V = \frac{88}{21} \times R^3$$

$$\text{Now, when} \quad R = \frac{3}{2}, \text{ we have}$$

$$V = \frac{88}{21} \times R^3$$

$$= \frac{88}{21} \times \frac{3 \times 3 \times 3}{2 \times 2 \times 2}$$

$$= \frac{99}{7}$$

$\therefore$ The volume of a sphere of radius $\frac{3}{2}$ cm, is $\frac{99}{7}$ cm ^{3}

4. Oscillations per minute of a pendulum vary inversely as the square root of its length. If oscillations of a pendulum of length 100 cm. are 60 per minute, find oscillations per minute if the length of a pendulum is 144 cm.

Let O stand for number of osicillations per minute, L stand for length of a pendulum.

$$O \propto \frac{1}{\sqrt{L}}$$

$$\therefore \quad O \times \sqrt{L} = k \qquad (k \text{ is a constant})$$

when $L = 100$, $O = 60$ we have

$$60 \times \sqrt{100} = k$$
$$\therefore \quad 60 \times 10 = k$$
$$\therefore \quad 600 = k$$
$$\therefore \quad O \times \sqrt{L} = 600$$

when $L = 144$, we have

$$O \times \sqrt{144} = 600$$
$$\therefore \quad O \times 12 = 600$$
$$\therefore \quad O = 50$$

$\therefore$ oscillations per minute are 50.

Miscellaneous Exercise - 10

1. If $x \propto y^3$ and $x = 16$ when $y = 2$, find (i) y when $x = 54$, (ii) x when $y = 4$.
2. x varies inversely as the cube of y and $y = 2$ when $x = \frac{2}{3}$. find y when $x = \frac{9}{4}$.
3. y varies inversely as the cube-root of x, and $y = 2$ when $x = \frac{1}{8}$. Find x when $y = 4$.
4. y varies directly as the square-root of x and $x = 0{\cdot}25$ when $y = 1{\cdot}5$. Find y when $x = 4$.
5. The area of a circle is proportional to the square of its radius. The area of a circle having radius 7 cm. is 154 sq. cm. Find the radius of the circle whose area is 1386 sq. cm.
6. The distance of the horizon from a place varies directly as the square-root of the height of the place. From a place 64 metres high the distance of the horizon appears to be 2 kilo-metres away. Find how far will the horizon appear from a height of 100 metres.
7. The force of attraction between two unlike magnetic poles varies inversely as the square of the distance between them. If the poles are 12 cm. apart, the force of attraction is 3 dynes. Find the force of attraction between two unlike magnetic poles when they are 8 cm. apart.

8. The cost of a diamond varies as the square of its weight. A diamond weighing 10 deci-grams costs Rs. 1600. Find the cost of a diamond of the same kind weighing 7 deci-grams.
9. The cost of a diamond varies as the square of its weight. A diamond weighing 12 deci-grams costs Rs. 2000. The diamond broke down into two pieces weighing 9 deci-grams and 3 deci-grams. What would be the loss to the merchant ?

● ● ● ●

Chapter - 11

Shares

§ 11.1

For a small business one person can invest sufficient capital. Capital required for a medium size business can be raised in a partnership. But when a large industry or a workshop is to be established, a partnership of a few persons is not able to provide the required capital. In such a case, some persons come together and float a company. These persons are known as **promoters** of the company. They divide the required capital into small parts called **shares**. These shares are generally worth Rs. 100 or Rs. 10 each. The capital raised by means of such shares is called the **Sharecapital.**

People who wish to invest their money in some company, consider the standing of the persons who are the promoters of the company, the objectives of the company and the programme of business of the company. They get shares of the company as a token of the sum of money invested. These people .who buy shares are called **share - holders** of the company and they are in a sense, owners of the company to the extent of the capital invested by them. For the management of the company, the share holders elect from among themselves a body of persons, which is known as the **Board of Directors** of the company.

As every share-holder of a company is an owner of the company (to the extent of his share - capital), naturally, he is entitled to a proportionate share of the profits of the company. The amount of profits on each share is called the **dividend**. The way in which this dividend is distributed, determines the two main types of shares : (1) preference shares and (2) equity shares.

1) Preference Shares :

At the end of the working year of a company, when accounts are finalised, the directors are able to ascertain the gross profit of the company. From this profit, certain provisions are to be made according to the Company Law. The amount of profit that remains after the provisions, is distributed among the share - holders. Share - holders who hold preference shares have a preferential claim over the profit as agreed to. The agreed rate of dividend to preference-share - holders remains the same, irrespective of whether the company makes any profits or whether it runs into loss. At present, preference shares generally fetch about 9 per cent of the dividend.

Sometimes, it may happen that a company incurs less or that there is not sufficient amount of profit after the provisions, to be distributed as dividend. In such circumstances, the company is not able to pay the agreed dividend to the preference - share - holders, that year. In that case even, the dividend on cumulative - preference shares goes on accumulating and is paid in prefence as and when the company can do so.

2) Equity Shares :

The profit of the company that is left over after its distribution to the preference-share-holders, is distributed among the equity - share holders. This dividend on equity shares may some-times be more or some-times less than the divided on the preference shares. If the company is doing good business, the equity shares may fetch proportionately much more than the preference shares. Also it is binding on the company under the law, not to pay dividend more than a certain percentage. In such circumstances, the residue profits are accumulated from year to year and then converted into share - capital which is distributed to equity - share - holders. These shares out of profits are known as **bonus shares.** Hence; people are prone to invest their money in equity *shares of the companies that are run on* sound basis and make sumptuous profit.

§ 11.2 Share Market (Stock Exchange)

The amount invested in shares of a company is never returned by the company. So whenever a share-holder needs money against his shares, he has to sell his share to somebody. This transaction, naturally, involves two persons - a buyer and a seller. The place where the transactions of buying and selling shares take place is known as the **share market** or **the stock exchange.** The buying and selling of shares cannot take place privately. It has to be done through a person or a body recognised by the government. Such a person is called a **share - broker**. A broker has got to be paid his charges for the transaction of buying and selling of shares, both by the purchaser and the seller. These charges are called **brokerage.**

The price of a share is printed on the share certificate. It is called the **face–value** of the share. If a company is making good profits or is likely to make good progress, the shares of such a company are in big demand in share-market. On the other hand, the shares of a company which runs into losses or which is not likely to show any signs of progress, lack such a demand. Consequently, shares of those companies which are in demand are sold at a price much higher than the face-value. The shares not in demand are sold at a price lower than the face value. The **selling price** or the **market price** of a share, therefore, is generally different from its face value and is dependent on the working of a company.

The company has nothing to do (or, is not concerned) with the price you pay for a share in the share-market. Form the point of view of a company, you have invested as much share-capital as the face-value of the shares only, and you are entitled to a dividend on the said face-value. In the same way, brokerage is also calculated on the face-value. It has nothing to do with the market-price.

Let us now get ourselves acquainted with the transactions of shares through examples.

§ 11.3 Example 1 :

The sum of money got by selling shares of Rs. 1300 in 8% at 155, was deposited in a bank at 7% per annum. What is the difference in the annual income ?

'8% at 155' means a share of face-value of Rs. 100 is being sold for Rs. 155 in the market and dividend at 8 per cent is paid (by the company) on it. When the face-value of a share is not stated it is conventionally taken to be Rs. 100. If otherwise, it should be clearly stated.

Also, since no mention of brokerage is made in the example. So if is not to be taken into consideration. When the same is mentioned, it has to be taken into account.

Let us now solve the example.

The market-price of a share of face value of Rs. 100 is Rs. 155.

∴ by selling shares of face-value of Rs. 1300, the sum of money

$$\text{realised} = 155 \times \frac{1300}{100} = \text{Rs. } 2015.$$

$$\text{The dividend on these shares} = \frac{1300}{100} \times 8 = \text{Rs. } 104$$

Now, the annual interest from the bank on Rs. 2015 at 7%

$$= 2015 \times \frac{7}{100} = \frac{14105}{100} = \text{Rs. } 141.05$$

∴ the income increases by 141.05 − 104 = Rs. 37.05

§ 11.4 Example 2 :

A sum of Rs. 1200 was invested in 5% shares at 78. When they rose to 84, all of them were sold. For each transaction brokerage at 2% was paid. What is the profit in the overall transaction ?

As there is no mention of the face value, it is to be taken at

Rs. 100 per share. Brokerage is mentioned. So, it should be taken into account.

A share of face value, of Rs. 100 is purchased at Rs. 78 and brokerage is paid at 2%.

$\therefore$ one share costs $78 + 2 =$ Rs. 80.

$\therefore$ for Rs. 1200, we get, $\frac{1200}{80} = 15$ shares.

At the time of selling each share fetched Rs. 84, but brokerage was paid at 2%. So, each share fetched $84 - 2 =$ Rs. 82 nett.

$\therefore$ total nett sum received $= 82 \times 15 =$ Rs. 1230.

$\therefore$ overall profit $= 1230 - 1200 =$ Rs. 30

11.5 Example 3 :

Two companies have shares of 5% at 80 and of 8% at 150 respectively. In which shares is it more profitable to invest our money ?

In dealing with such examples, if is better to suppose that equal sums of money are invested in both kinds of shares. Then it becomes easy to determine from the dividends, which share is more profitable. To make calculations easier it is advisable to invest as much sum of money as the market-price of one of the two kinds of shares.

Let us now solve the example.

Suppose, that a sum of Rs. 80 was invested in each of the two kinds of shares.

$\therefore$ We would get Rs. 5 as dividend from the first kind of shares.

Now, the dividend from the investment of Rs. 80 in 8% at 150

$$= 80 \times \frac{8}{150} = \frac{640}{150} = \frac{64}{15} \text{ Rs. } 4\frac{4}{15}$$

$\therefore$ It is more profitable to invest in 5% at 80.

From the above examples, you must have grasped the language used in examples on shares and also the method of tackling the examples. Now solve the following examples for practice.

Miscellaneous Exercise - 11

1. The sum of money obtained by selling 5% shares at 96, of the face value of Rs. 2,800, was invested in 6% at 112. What is the difference in the annual income ?
2. A man invested Rs. 4960 in 8% shares at 124. How much dividend will he get ? How much per cent of dividend does he get on his investment ?
3. A man purchased shares of face value of Rs. 2400 by investing Rs. 3000. What was the market price of a share ? If the shares fetched 8% dividend what percentage of dividend did he get on his investment ?
4. Suratwala spent Rs. 2610 in purchasing 9% shares at 140. After getting the dividend on them, he sold them at the same price. In each transaction he had to pay 5% brokerage. Did he gain or lose in the total transaction ? By how much ?
5. Hakimbhai and Adranwala each invested Rs. 1820 in $5\frac{1}{2}$% at 91 and in $7\frac{1}{2}$% at 130 respectively. Whose investment is more profitable and by how much ?
6. Subhash invested Rs. 912 in shares of face value of Rs. 50 at 56. After receiving dividend on them at 9% he sold them at 55. In each of the transactions he paid 2 % brokerage. How much did he gain or lose in the overall transaction?
7. Two companies have shares of 7 % at 116 and 9 % at 145 respectively. In which of the shares would the investment be more profitable ?
8. A man invested Rs 11622 in 6% shares at 104 and Rs. 11000 in 10.40% shares at 143. How much income would he get in all ?
9. The amount realised by selling 7% shares at 119 of the face value of Rs 2,000 was invested in 5% shares at 85. What would be the difference in his annual income ?
10. A man purchased shares worth Rs. 12000 when the market price was Rs. 92.50. Out of those shares he sold shares of face value Rs. 7200 when the market rate was 102, and sold the remaining shares at 96. He had to pay 2.5% brokerage each time. What was his gain or loss on the whole ?

● ● ● ●

Chapter 12

Statistics

§ 12.1

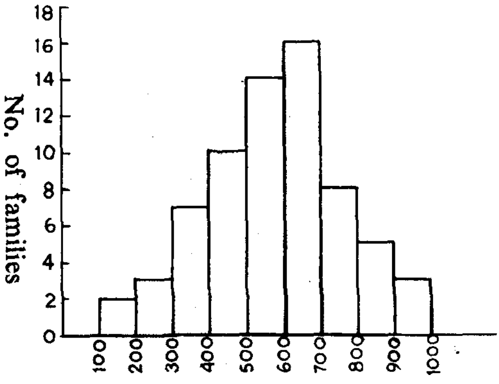
The science of statistics is a powerful mean of studying groups, big or small. Knowledge of statistics is also widely useful in the study of sciences, not only like Physics and Chemistry, but also in the study of agricultural science, medicine, social sciences, psychology, education and even in economics.

The information or data that we collect about a particular group is mostly numerical in nature. Mere collection of information does not throw much light upon the nature of the group as a whole. Such material or information is called ' Raw numerical data. When this data is subjected to analysis many aspects regarding the nature of the group become clearer. But before any analysis can be made the first thing we have to do is to classify this raw data.

After careful observation of the raw data, suitable class interval is first fixed. After deciding the number of observations that fall in a given class interval we obtain a frequency distribution giving frequency in each class interval. A visual representation is obtained if a histogram or a frequency polygon is drawn from the information supplied in the frequency table. All this you have learnt in your IXth standard.

Let us, however, make a revision of the same.

Below is drawn a histogram giving details about the monthly income of some families in a village. Answer the following questions with the help of the histogram.



Monthly income of a family (Rs.)
Fig. 12.1

(1) What is the length of each class interval ?

(2) Prepare the frequency distribution table with the help of the histogram.

(3) Which class interval accommodates the maximum number of families ?

(4) Find out the class interval whose frequency is 7.

(5) Find out the total number of families taken into consideration.

(6) Prepare the frequency polygon with the help of the histogram.

§ 12.2

The classification of the data gives a compact form to the otherwise widely spread numerical items in the raw data. It also unfolds to some extent the structure of the group. But to further clarify the nature of the given group and also to be able to compare the given group with another similar group, it is found useful to render the numerical data a compact form. To be able to choose a single characteristic number which will represent the group as a whole is the second important step in the analysis of a given numerical data. We will mostly concentrate on this second step in our study of statistics this year.

To illustrate the point made in the above paragraph let us study some examples.

(1) The people of Maharashtra are 165 cm tall.

(2) Everyday I walk 6 kilometers.

(3) A sack of sugar weighs 100 kilograms.

(1) When we say that the people of Maharashtra are 165 cm tall we do not mean that each and every man from Maharashtra is exactly 165 cm tall. All we mean by this statement is that if a large crowd of Maharashtrians is considered then most men from this group will have height near about 165 cm In other words we can say that the number 165 cm is the most appropriate number which represents the height of the people of Maharashtra as a whole.

(2) Everyday I have to go to my office. The office is roughly 3 km. from my house. Hence everyday to and fro journey to the office and some incidental walk amounts to about 6 km. Therefore, I say that everyday I walk 6 km.

(3) If many sacks of sugar are weighed it is found that a sack weighs round about 100 kg. So in this case the statement that a sack of sugar weighs 100 kg. makes sense.

We should remember that we could easily decide on the representative numbers of the above groups because the individual items of those groups were within the range of our everyday knowledge and experience. From examples considered above we observe that every numerical raw data regarding some group contains two important properties as stated below.

(1) Numerical data is spread between two numbers, one which is the largest and the other which is the smallest.

(2) Most numerical values in the given data cluster round some number.

Let us clarify this further by considering the following example.

There were 20 sacks of sugar in a godown. Each sack was weighed. The weights in kilogram were as follows :

102.5, 100, 99, 101, 100, 103,

98.5, 99, 100, 101, 100, 101.5,

99.5, 97, 100.5, 98.5, 104, 101,

99.5, 100.5.

Let us plot points representing the above numbers on a graph paper.

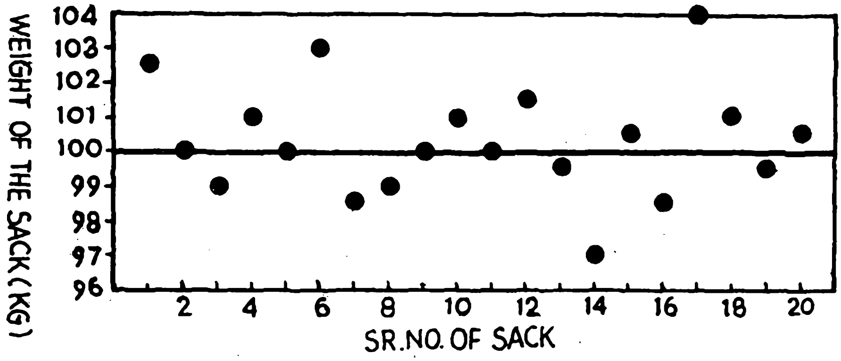


Fig. 12.2

The graph clearly shows that,

(1) The maximum and the minimum weights are 104 kg. and 97 kg. respectively.

(2) Weights of the most of the sacks lie near about 100 kg. (As many as 13 sacks out of 20 weigh between 99 kg. to 101 kg.) Therefore the number 100 represents well the group of weights given in the example.

Thus when we collect a numerical data regarding a particular group, we find that there is always a number about which most of the numbers from the given data seem to crowd.

This tendency of a group is called its **central tendency.**

§ 12.3

To compare two given groups their central tendencies are measured. There are different methods of measuring central tendencies of a group. Each method has its own merits and demerits. We have to select a suitable method. Keeping in view our main purpose of study this year we are going to make a study of the following three types of central tendencies.

(1) Arithmetic mean (2) Median (3) Mode.

1 Arithmetic Mean : This measure of central tendency has to be calculated from a given numberical data. The arithmetic average of all numbers in the data is called its Arithmetic Mean. To evaluate the Arithmetic Mean we have to divide the sum of all numbers in the numerical data by the total number of items.

Therefore, if N items in a given data are as follows :

$x_1, x_2, x_3, \ldots \quad x_n,$

its mean will be the number given by

$$\frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{N.}$$

If we represent mean by $\overline{x}$ and the sum $(x_1 + x_2 + x_3 + \ldots + x_n)$ by Σx_i (Σ is read as sigma), then.

$$\boxed{\overline{x} = \frac{\Sigma x_i}{N}}$$

Example (1) :

Consider example in § 12.2. Find the mean weight of 20 sacks of sugar.

$$\bar{x} = \frac{\Sigma x_i}{N}$$

$$= \frac{\begin{bmatrix} 102.5 + 100.0 + 99.0 + 101.0 + 100.0 + 103.0 \\ + 98.5 + 99.0 + 100.0 + 101.0 + 100.0 + 101.5 \\ + 99.5 + 97.0 + 100.5 + 98.5 + 104.0 + 101.0 \\ + 99.5 + 100.5 \end{bmatrix}}{20}$$

$$= \frac{2006}{20}$$

$$= 100.3 \text{ kg.}$$

We observe that the answer is consistent with the answer we found from the graph of the given data.

In the above example addition of 20 numbers was quite labourious. This labour can be saved if we adopt the method as shown in the following example.

Example (2) : Find the arithmetic mean of 45, 47, 50, 48 :

Method	Activity
(1) Observe the data carefully and assume a suitable number as ' mean '. Call it A	(1) 45 and 50 are the minimum and the maximum values respectively. The number 48 is midway between them. So assume 48 as mean. $\therefore \quad A = 48$
(2) Subtract the assumed mean from each number in the data represented. The results of subtractions are denoted by x'	(2) $45 - 48 = -3 = x'_1$ $47 - 48 = -1 = x'_2$ $50 - 48 = 2 = x'_3$ $48 - 48 = 0 = x'_4$

(3) Divide the sum $\Sigma x'_i$ by N.	(3) $\frac{\Sigma x'_i}{N} = \frac{-3-1+2+0}{4}$ $= \frac{-2}{4}$ $= -0{\cdot}5$
(4) Add $\frac{\Sigma x'_i}{N}$ to the assumed mean A. The answer will give you the real mean, that is x	(4) $x = A + \frac{\Sigma x'_i}{N}$ $= 48 + (-0.5)$ $= 47.5$

This example incidentally tells us the formula for determining the arithmetic mean of a given data.

$$\boxed{\bar{x} = A + \frac{\Sigma x'_i}{N}}$$

This method is called the 'method of assumed mean'

(1) If you add all the numbers and divide the sum by 4, you will get the same answer, 47·5

(2) If you assume some other number as the mean still you will get the same answer.

In this example we have dealt with only four numbers. But this method need not be used when only a few numbers are in the given data. This method proves useful, when there is a large number of items in the data. This will be illustrated further by considering the following example.

Example (3) :

Find out mean weight of 20 sacks of sugar, considered in example (1), by using the method of assumed mean.

The weights of the sacks range from 97 to 104 kilograms. So, let us take 100 kilograms as the assumed mean.

Let us prepare a table as per the method explained above in example (2).

Total number, $N = 20$
Assumed mean, $A = 100$

Numbers that are greater than A			Numbers that are less than A		
Sr. No.	x_i	$x_i - A = x'_i$	Sr. No.	x_i	$x_i - A = x'_i$
1	102.5	2.5	1	99.0	− 1.0
2	101.0	1.0	2	98.5	− 1.5
3	103.0	3.0	3	99.0	− 1.0
4	101.0	1.0	4	99.5	− 0.5
5	101.5	1.5	5	98.5	− 1.5
6	100.5	0.5	6	97.0	− 3.0
7	104.0	4.0	7	99.5	− 0.5
8	101.0	1.0			
9	100.5	0.5			
	Total	15.0		Total	− 9.0

$$\therefore \quad \bar{x} = A + \frac{\Sigma x'_i}{N}$$
$$= 100 + \frac{15 + (-9)}{20}$$
$$= 100 + \frac{6}{20}$$
$$= 100 + 0.3$$
$$= 100.3$$

Note : Here 100 kg. was taken as assumed mean. Because we observed that four sacks have weight exactly 100 kg. for, those four observations we have 0 for $x_i - A$. For this reason it was not shown in the above table.

$\therefore$ Mean weight of sacks = 100.3 kg.

Some–times the items in the data are repeated. In such cases the method of calculating the mean changes slightly.

Study the following example.

Example (4) :

40 students in a class had for their pocket expenses the following amounts in rupees, when they started for an excursion.

150	200	200	100	150	100	300	150
250	200	250	150	300	100	350	150
200	100	250	300	250	250	100	300
150	150	200	300	350	100	50	350
200	150	200	200	200	450	250	200

Find the arithmetic mean of the above sums of money.

Observing the given data we infer as,

(1) All numbers are the multiples of 50

(2) The least number is 50 and the greatest number is 450

(3) There are 10 students bringing the amount of Rs. 200.

Considering these observations let us prepare the frequency distribution table.

Amount in Rs, (x)	No. of students bringing the amount (f)	$f \times x$
50	1	50
100	6	600
150	8	1,200
200	10	2,000
250	6	1,500
300	5	1,500
350	3	1,050
400	0	—
450	1	450
Total	40	8,350

[The principle used here to sum up the numbers is $x + x + \dots\dots + x$ (f times) $= f \times x$]

$\therefore$ mean $\bar{x}$

$$= \frac{\text{total amount } (\Sigma fx)}{\text{total number of students } (N)}$$

$$= \frac{8350}{40}$$

$$= 208.75$$

The formula used in this example is

$$\boxed{\bar{x} = \frac{\Sigma f.x}{N}}$$

Example 5 :

If this example be solved by using the method of assumed mean, the calculations will be seen to be minimised.

We observe that maximum number of students are bringing Rs. 200. So let 200 be the assumed mean.

Since all the amounts are the multiples of Rs. 50. we will express the difference between the observations and the assumed mean in units of 50. We denote this unit by j.

For example, $50 - 200 = -150 = -3 \times 50$

$\therefore\ 50 - 200 = -3$ units.

Similarly. $300 - 200 = 100 = 2 \times 50$

$\therefore\ 300 - 200 = 2$ units.

Now let us prepare the table.

Amount (Rs,) (x)	Frequency (f)	Unit difference beween the numbers and the assumed mean (x')	$f \times x'$	
50	1	−3	−3	
100	6	−2	−12	
150	8	−1	−8	−23
200	10	0	0	
250	6	1	6	
300	5	2	10	
350	3	3	9	
400	0	4	0	
450	1	5	5	30
	40			7

In this example $N = 40$, $A = 200$,

$$\Sigma f.x' = -23 + 30$$

$$= 7 \text{ units}$$

$$= 7j$$

$$\therefore \text{ Mean. } \bar{x} = A + \frac{\Sigma f.x' \text{ units}}{N}$$

$$= A + \frac{\Sigma f.x'\ j}{N}$$

$$= 200 + \frac{7 \times 50}{40}$$

$$= 200 + 8{\cdot}75$$

$$= 208{\cdot}75$$

The formula used in this method was

$$\boxed{\bar{x} = A + \frac{\Sigma\, f{\cdot}x'}{N} \cdot \text{j}}$$

In this case also you may verify that even if the number we take for the assumed mean is changed, the real mean does not change.

We have used a few simple mathematical techniques. Study them carefully.This will help you to understand why these methods are used.

So far we have studied a few methods of finding the arithmetic mean. We should adopt the method that is suitable to a given problem.

To get the arithmetic mean we take into account all the numerical items of the data. Hence it is the most reliable and valid measure of the central tendency of the given group. Yet the arithmetic mean has its own limitation. The follwing example will bring to your notice this limitation

Let us say that 12, 10, 8, 9, 66, are the marks obtained by five students in a test.

The arithmetic mean of the marks is

$$\frac{12 + 10 + 8 + 9 + 66}{5} = \frac{105}{5} = 21.$$

Now if we observe the data carefully, we find out that the marks obtaind by four students of the five are in the vicinity of the number 10 only. The fifth student has marks 66. The arithmetic mean of these four numbers is 9.75.

Now the arithmetic mean of the four numbers of the given five is closer to 10. Obviously the number 21 does not represent the group in the real sense. The reason is obvious. The difference between 66 and the number 10, about which the marks obtained by the rest of students lie, is very large.

In short we can conclude that if the given group is such that the scores of a few numerical items in the data differ greatly from those of its majority, arithmetic mean does not adequately represent the group.

Median : when the numbers in the raw numerical data are arranged in ascending or descending order, the number which comes exactly in the middle of this arrangement is called the median of the group.

To determine the median no calculation is necessary. You will easily notice that extreme values do not affect the median.

Example (1):

Find the median of the following raw material,

5, 11, 6, 48, 15, 10, 6, 13, 10.

First we shall arrange the numbers in the ascending order.

5, 6, 6, 10, 10, 11, 13, 15, 48.

There are nine numbers in the raw material. So fifth number in the ascending order is the median.

$\therefore$ median $= 10$

(It should be noted that the number 48 in the given raw data is bigger in comparison with other numbers, but it does not affect the median.)

Example (2):

Find the median,

75, 98, 72, 105, 101, 78, 75, 85.

Arrange the given numbers in the ascending order,

72, 75, 75, 78, 85, 98, 101, 105.

In all there are eight numbers. That is the number of numbers is even. So it cannot be said that one and only one number is in the middle. Both fourth and fifth numbers are in the middle. In such case the average of two such numbers is taken to be the median.

$$\therefore \quad \text{median} = \frac{78 + 85}{2} = \frac{163}{2} \quad 81{\cdot}5$$

Generally if the numbers arranged in the ascending order are n,then

(1) if n is odd the median of the group is the number that is at $\left(\frac{n+1}{2}\right)^{th}$ place.

(2) if n is even the average of numbers that are at $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n+2}{2}\right)^{th}$ place is the median of that group.

Following example illustrates how median is to be determined when the raw material is given in a classified form.

Example (3) : Find the median of the population from the frequency table.

Number	Frequency
10	1
20	3
30	7
40	10
50	13
60	8
70	6
80	5
90	5
100	2

Cumulative Frequency
1
1 + 3 = 4
4 + 7 = 11
11 + 10 = 21
21 + 13 = 34
34 + 8 = 42
42 + 6 = 48
48 + 5 = 53
53 + 5 = 58
58 + 2 = 60

Cumlative frequency table has been prepared as shown in the column adjoining the frequency column. From it is seen that there are in all 60 numbers whose median is to be determined.

$\therefore \quad N = 60$

As N is even, the median will be the average of the numbers that fall at 30th and 31st place. As the numbers are arranged in ascending order 30th and 31st numbers can be fixed from the table.

From the column of cumulative frequency it is seen that both 30th and 31st numbers are 50 and 50 each.

$$\therefore \quad \text{median} = \frac{50 + 50}{2} = 50.$$

Mode :- It is also an important measure of central tendency. Zizek has defined the mode as follows:

‘The value occuring most frequently in a series (or group) of items and around which the other items are distributed most densely, is the **Mode** of the distribution’

Example 1 :

Ages of 50 students from standard X are recorded as below. Find the mode of ages of the pupils,

16	15	15	17	15	14	17	16	15	15
17	15	18	14	15	16	16	17	16	15
15	14	17	15	16	15	15	15	16	15
15	16	16	14	15	15	14	17	18	16
16	15	16	17	15	16	15	17	15	15

First let us prepare frequency table.

Age, (yrs,)	Frequency
14	5
15	22
16	13
17	8
18	2
Total	50

From the table it is seen that there are 22 students having age 15. Thus 15 occures most number of times. So in this case 15 is the mode of ages of students in standard X.

We say that a student of standard X is of age 15.

Mode as a measure of central tendency has its uses mostly in industry. From mode one can know best about the choice of people in regard to colour, size and value of things. From it industrialist can determine the type of production and traders can decide about the type of material to be produced. For example, if majority of people prefer black umbrella, the industrialist will manufacture black umbrellas and trader will place their demands only for black umbrellas.

Miscellaneous Exercise - 12

(1) Find the mean from the material given below,

(i) 5, 13, 10, 8, 7 (ii) 478, 517, 550, 522, 483.

(2) Find the mean of numbers given in example (1) by the method of assumed mean.

(3) Find the median of the numbers given below,

(i) 13, 22, 17, 15, 13, 16, 30.

(ii) 75, 84, 78, 95, 100, 76, 68, 84, 69, 78.

(4) 25 students appeared for a competitive examination. Their individual scores are given below,

98 92 100 97 98 81 98 95 87

98 94 98 91 100 92 98 82 94

98 85 92 93 99 98 94.

Find the median and the mode of the scores.

(5) In a test 10 pupils scored as follows,

78, 82, 90, 87, 80, 84, 75, 90, 88, 84.

(i) Taking 80 as the assumed mean, find the true mean of the scores.

(ii) Taking 85 as the assumed mean, find the true mean of the scores.

(6) Yield per acre got by Mahipatrao and Hambirrao from year 1971 to 1980 is given.

By obtaining the mean of the yields, determine which of the lands is more producivet. (Numbers in quintal.)

Year	Mahipatrao	Hambirrao
1971	40	43
1972	47	45
1973	42	41
1974	50	47
1975	48	44
1976	47	50
1977	51	47
1978	45	47
1979	55	51
1980	48	49

(7) Rates of tickets for a dancing performance were Rs. 10, 20, 50, 100 and 200. The adjoining table shows the sale of each kind of ticket.

From the table find the median.

Rate (Rs.)	Frequency
10	1150
20	475
50	200
100	125
200	50

(8) The following table shows the donations received in aid of a school for blinds.

Amount of donation(Rs.)	10	20	30	40	50
Number of doners	42	28	7	5	18

(i) Taking 20 as the assumed mean find the exact mean.

(ii) Find the median of the material given.

● ● ● ●

ANSWERS

Chapter 1

Exercise 1.1

1. (1) 6. (2) 2^7. (3) -7. (4) 7. (5) 5.
(6) 1. (7) -8. (8) 16. (9) 0, 1, (10) -4.
(11) 10. (12) 3. (13) -4. (14) -8. (15) -3.
(16) 4. (17) 3. (18) -6.

2. (i) $(81)^{\frac{3}{4}} < (27)^{\frac{4}{3}}$. (ii) $2^{-4} = 4^{-2}$. (iii) $(\frac{9}{16})^{\frac{3}{2}} > (\frac{8}{27})^{\frac{4}{3}}$

3. (i) 25. (ii) 8. (iii) 27. (iv) $128\sqrt{2}$.

Exercise 1.2

1. (i) 8th root of 5. (ii) 14th root of 10. (iii) 6th root of $\frac{4}{3}$

2. (i) $5^{\frac{1}{3}}$ or $\sqrt[3]{5}$. (ii) $48^{\frac{1}{12}}$ or $\sqrt[12]{48}$.

3. (i) 4. (ii) 21. (iii) $\frac{4}{7}$.

Exercise 1.3

1. (1) 2, 25. (2) $\frac{7}{3}$. (3) $\frac{5}{7}$. (4) $\frac{3}{5}$. (5) 3.
(6) $\frac{5}{3}$. (7) $-\frac{1}{4}$. (8) $-\frac{5}{7}$. (9) $-\frac{11}{9}$. (10) $-\frac{6}{5}$.
(11) $-\frac{1}{2}$. (12) $\frac{3}{8}$. (13) $-\frac{5}{6}$. (14) $-\frac{3}{28}$. (15) 0, 1.
(16) 4, 4.

2. (1) $\frac{1}{64}$ (2) $\frac{1}{729}$ (3) 4. (4) $\frac{1}{256}$.

3. (1) 1. (2) 1. (3) 625. (4) 512.

4. (1) $\frac{1}{3}$. (2) -2. (3) -2. (4) $\frac{7}{9}$.
(5) $-\frac{1}{7}$. (6) $-\frac{2}{5}$.

Exercise 1.4

1. (1) $\frac{3}{7}$. (2) 1. (3) -1. (4) $\frac{1}{3}$. (5) $\frac{1}{6}$.
(6) $\frac{9}{4}$. (7) $-\frac{7}{6}$. (8) -3. (9) 1. (10) $-\frac{7}{10}$.
(11) $-\frac{1}{8}$. (12) $-\frac{1}{3}$. (13) $-\frac{13}{25}$. (14) $-\frac{2}{3}$.

2. (1) $\frac{1}{16}$. (2) $\frac{1}{3}$. (3) $\frac{1}{100,000}$. (4) 19,53,125.

3. (1) 25. (2) $\frac{1}{32}$. (3) $\frac{1}{27}$. (4) 8,192.

4 (1) $\frac{1}{2}$. (2) -2 (3) $-\frac{3}{8}$. (4) 0.

Exercise 1.5

1. (1) $\frac{4}{5}$ (2) $-\frac{2}{3}$ (3) $\frac{3}{14}$ (4) $-\frac{9}{56}$ (5) $-\frac{16}{7}$ (6) $-\frac{3}{2}$

2. (1) $\frac{1}{512}$ (2) $\frac{1}{15625}$ (3) 1024 (4) 343.

3. (1) 8. (2) $\frac{1}{3}$. (3) 1.
4. (1) $\frac{1}{2}$. (2) -2 (3) $\frac{1}{6}$ (4) $-\frac{8}{5}$

Exercise 1.6

1. (1) $\frac{3}{4}, \frac{3}{4}$. (2) $-\frac{8}{3}, -\frac{8}{3}$ (3) $-\frac{7}{2}, -\frac{7}{2}$.
2. (1) $\frac{4}{5}$. (2) $\frac{1}{16}$.

Miscellanous Exercise - 1

1. (1) $\frac{14}{3}$ (2) $\frac{1}{5}, -\frac{6}{5}$ (3) -1 (4) $-\frac{9}{7}, \frac{20}{7}$
2. (1) 1. (2) 6. (3) 36. (4) $\frac{1}{800}$. (5) 4.
3. (1) $x = -\frac{17}{16}$, $y = \frac{1}{16}$, $z = \frac{1}{8}$.
(2) $x = -\frac{3}{5}$, $y = \frac{14}{9}$, $z = \frac{11}{6}$.
4. (1) $x = -2$. (2) $x = \frac{11}{5}$. (3) $x = \frac{11}{3}$

Chapter 2

Exercise 2.1

1. (i) 9:14. (ii) 7:5. (iii) 2:1. (iv) 4:3
(v) 3:2 (vi) 3:4 (vii) $\sqrt{3} : \sqrt{5}$ (viii) 1:3 (ix) 5 : 4
2. (i) 3:5. (ii) 11:40. (iii) 7:100. (iv) 2:21. (v) 3:4 (vi) 5:4
3. 30 years, 12 years.
4. 28, 49.
5. 60, 25.
6. 7 cm., 4.2 cm.
7. 60°, 80°, 40°.
8. 60°, 80°, 100°, 120°.
9. 2:3, 4:9.
10. 2:5, 2:5.

Exercise 2.2

(1) > (2) > (3) = (4) > (5) < (6) = (7) =
(8) < (9) < (10) > (11) < (12) = (13) < (14) < (15) >.

Exercise 2.3

1. 41. 2. $\frac{13}{7}$. 3. $\frac{5}{2}$. 4. $\frac{+1}{9}$. 5. 0.
6. (1) $x = 18$ or $x = \frac{1}{2}$. (2) $x = 6$. (3) $x = 3$.
(4) $x = 0$ (5) $x = 0$ (6) $x = \frac{10}{3}$

Exercise 2.4

1. (1) 11, 4. (2) 24, 5, $x + 2y \div 4z$.
   (3) $2l + 3m - 4n$, $5x - 2y + z$. (4) $x + y$.
   (5) $m - l$, $2(m - n)$.

Exercise 2.5

1. (1) 21. (2) 15. (3) $\frac{c^2d}{a}$. (4) $\frac{y^3}{x}$.
2. (1) 15 (2) 99. (3) x^3y^2. (4) xz.
3. (1) 28. (2) 10. (3) $10\sqrt{10}$ (4) a. (5) $\frac{p+q}{pq}$.
4. (1) 36. (2) 16. (3) $\frac{a+b}{a-b}$ (4) 2.
5. 3.
6. 4.
7. 3, 27.

Misellaneous Exercise - 2

1. (1) 5:8. (2) greater (3) less (4) 5:3. (5) $\frac{5}{2}$
   (6) $\frac{11}{3}$. (7) $\frac{6}{5}$ (8) (51 − 39).
   (9) 18. (10) 24.
2. (1) equality (2) 75. (3) No effect. (4) $x^2 - y^2$.
   (5) The ratio of present ages will be greater than the ratio of the ages some years hence.
3. (1) $\frac{7}{5}$. (2) 3:2.
4. 12 years and 10 years.
5. 48 years and 20 years.
6. 72 and 45.
7. Rs. 2,400 Rs. 4,000 Rs. 6,400

Chapter 3

Exercise 3.1

1. Rs. 3,296·30 2. Rs. 712·08, 10·4% 3. Increase, 4%
4. 21·5%, Rs. 1,020·60 5. 21·8%, 609. 6. 62% Rs. 46,170
7. Rs. 1300 8. Two lacs of rupees. 9. Rs. 4,000 10. Rs. 29,200

Exercise 3.2

1. 1,875 quintals 2. Rs. 16,000 3. Rs. 10,000 4. Rs. 4,800

Exercise 3.3

1. 8,400. 2. 800. 3. 273. 4. 1,001.

Exercise 3.4

1. 200 grams. 2. 20 litres, 30 litres. 3. 240 grams, 160 grams.
4. 12.5 grams.

Miscellaneous Exercise - 3

1. 17·6% 2. 112, 3·2% 3. 26,508. 4. 300.
5. one lac. 6. Rs. 60,000 7. 225 liters. 8. 125 kilograms each.
9. Rs. 2,600 10. 1,107 hectares.

Chapter 4

Exercise 4.1

1. Rs. 95 and Rs. 133 2. Rs. 8,000 Rs. 9,600 and Rs. 11,200
3. Rs. 360, Rs. 450 and Rs. 540 4. Rs. 540 Rs. 420 and Rs. 720.

Exercise 4.2

1. Rs. 600 Rs. 1,500 and Rs. 1800 2. Rs. 2,800 and Rs. 2,000
3. Rs. 1,104 Rs.2,484 and Rs. 3,312 4. Rs.1,700 Rs.1,775 and Rs.1,850.

Exercise 4.3

1. Rs. 540 and Rs. 450 2. Rs. 180 3. Rs.720 and Rs. 600
4. Rs. 2,700 5. Rs. 945.

Miscellaneous Exercise - 4

1. Rs. 900 and Rs. 540 2. Rs. 1,196 Rs. 1,495 and Rs.1,794
3. Rs. 8,750 and Rs. 12,250 4. Rs. 4,550 Rs.8,592
5. Rs.337.50 Rs.360 and Rs.367.50
6. Rs. 6,000 7. 4 months.
8. Rs. 16,000 9. 6 months.
10. 5 months and 7 months.

Chapter 5

Exercise 5.1

1.

(1)	Point	Coordinates	(2)	Quadrant or Axis
	T	$(4, 0)$		X - axis
	M	$(4, 4)$		I
	Q	$(-5, -3)$		III
	P	$(4, 2)$		I
	A	$(-4, 2)$		II
	B	$(-5, 0)$		X - axis
	U	$(0, -3)$		Y - axis
	V	$(4, -5)$		IV

3. $(3, 4)$, $(1{\cdot}5, 2)$.................... I
$(-2, 5)$, $(-\sqrt{3}, \sqrt{2})$......... II
$(-3, -5)$, $(-\sqrt{2}, -\sqrt{3})$......... III
$(6, -3)$, $(\sqrt{2}, -3)$, $(1{\cdot}5, -2{\cdot}3)$ .. IV
$(-7, 0)$, $(-\sqrt{3}, 0)$ $(0, 0)$........... X - axis
$(0, 2)$, $(0, -5)$, $(0, 0)$............Y - axis
[$(0, 0)$ lies on both axes.]

4. (1) y (2) second (3) third (4) Y (5) $(0, 0)$

5. (1) x - coordinate 4, parallel to Y - axis, x - coordinate 4
(2) parallel to x axis, y - coordinate, -3

Exercise 5.2

1. Points are collinear. $y = 5$

2. $x = -3$, parallel to Y - axis

3. (1) equation of line AB is $y = 0$, equation of line PQ is $y = 2$, equation of line QS is $x = -4$, equation of line RS is $y = -5$
(2) line MN (3) $(5, -3)$ (4) $y = -5$

4. (1) $x = 4$ (2) $y = -2$ (3) $y = 5$ (4) $x = -4$

5. (1) $(4-2)$ (2) $(-3, 2)$ (3) $(0, -5)$
(4) $(0, 0)$ (5) $(-3, 0)$ (6) $(-3, -5)$

Exercise 5.3

1. (1) $y = 2x$, (3) $3y = x$, (4) $y - 2x = 0$, (5) $2y = 3x$ lines will make acute angle will the positive direction of X-axis because their slopes are positive.

   (2) The line $y = -\frac{1}{3}x$ will make obtuse angle with the positive direction of the X - aixs because its slope is negative

   (6) The line $x = 5$ will make a right angle with the positive direction of the X – axis because it is parallel to Y- axis.

2 The lines (1) $x = 5$, (3) $x = 0$, and (5) $x = 0 \cdot y$, have no slope, and slope of the lines (2) $y = -4$, (4) $y = 4$, is zero.

3. $a = -4$, $b = 0$, $c = 3$ and $d = -5$

4. Slope $= \frac{1}{2}$, passes through the origin, $y = \frac{1}{2}x$

5. (1) $(5, 5)$ (2) passes through the origin (3) slope $= 1$, $y = x$
   (4) point $(-4, 4)$ lies on the line PQ. $y = -x$ (5) $y = 2x$
   (6) $a = 3$ (7) lie on the line CD. $y = -\frac{1}{2}x$ (8) (a) $y = 2x$
   (b) $y = -\frac{1}{2}x$ (c) $y = x$ (d) $y = -x$.
   order : (c), (a), (d), (b)

Exercise 5.4

1.

No.	Equation of line	Slope	Y-intercept
(1)	$y = x$	1	0
(2)	$y = x + 4$	1	4
(3)	$y = x - 3$	1	-3

The above lines are parallel to each other.

2. $y = 2x - 4$

3. The lines $y = -x + 3$, $x + y = 7$, $2x + 2y = 5$, are parallel to $y = -x$

5.

No.	Slope-Intercept form	Slope	Y-intercept
(1)	$y = \frac{4}{5}x - \frac{3}{5}$	$\frac{4}{5}$	$-\frac{3}{5}$
(2)	$y = -x + 5$	-1	5
(3)	$y = -\frac{1}{5}x + \frac{7}{5}$	$-\frac{1}{5}$	$\frac{7}{5}$
(4)	$y = 0.x + \frac{4}{3}$	0	$\frac{4}{3}$

6.

No	X – intercept	Y–intercept
(1)	3	2
(2)	16	12
(3)	2	-5
(4)	-6	4

7. $a = 1$ 8. $A = 2$ 9. X – intercept a and Y – intercept b, slope $= -\frac{a}{b}$
10. slope $= -\frac{1}{2}$; $y - 3 = -\frac{1}{2}(x - 4)$ 11. $h = 2$ 12. Lines are perpendicular to each other because product of their slopes is -1.
14. slope $= -1$, $y = -x + 6$ 15. are perpendicular. $(3, 2)$
16. (1) slope $= -\frac{1}{2}$, Y – intercept $= 2$ (2) $y = -\frac{1}{2}x + 2$
(3) $y = x - 1$ (4) $(2, 1)$

Exercise 5.5

(1) $x = 3$, $y = 2$ (2) $x = 5$, $y = 2$ (3) $x = 0$, $y = -3$
(4) $x = -2$, $y = 4$ (5) $x = 4$, $y = 1$ (6) no solution.

Miscellaneous Exercise – 5

2. are collinear. $y = x + 3$ 3. slopes are equal. Yes.
4. $y = -\frac{1}{2}x + 4$ 5. (1) $y = \frac{3}{2}x - \frac{5}{2}$ (2) $\frac{x}{\left(\frac{5}{3}\right)} + \frac{y}{\left(-\frac{5}{2}\right)} = 1$
6. slope $= \frac{3}{2}$, X – intercept $= \frac{5}{3}$, Y–intercept $= -\frac{5}{2}$
7. lies on the line because the coordinates satisfy the equation.
8. $y - 1 = 3(x - 2)$ 9. $-\frac{1}{2}$ 10. slope $= -\frac{1}{3}$; $y - 2 = -\frac{1}{3}(x - 0)$
11. (1) $y = 2x$ (2) $y - 3x = 0$ (3) $y - 4 = 2(x - 2)$ (5) $x = 0$

12. $A = 2$ and $B = 3$ 13. $A = 2$ and $B = 6$ 14. $x = -1$, $y = 1$

15. X – intercept $= \frac{10}{3}$, Y – intercept $= -2$ 16. $c = 10$; $(0, -\frac{5}{2})$

17. $\frac{x}{3} + \frac{y}{-2} = 1$, slope $= \frac{2}{3}$ 18. slope $= \frac{3}{2}$; $y = \frac{8}{2}x + 3$

19. $(7, 9)$ 20. $a = -2$

Chapter 6

Exercise 6.1

(1) $\{4\}$ (2) $\{3\}$ (3) $\{1\}$ (4) $\{14\}$ (5) $\{-5\}$
(6) $\{1\}$ (7) $\{-1\}$ (8) $\{16\}$ (9) $\{0\}$ (10) $\{10\}$
(11) $\{1\}$ (12) $\{4\}$ (13) $\{8\}$ (14) $\{-2\}$ (15) $\{4\}$
(16) $\{10\}$ (17) $\{9\}$ (18) $\{-11\}$ (19) $\{-\frac{52}{19}\}$ (20) $\{\frac{13}{4}\}$

Exercise 6.2

(1) $x = 7$ or $x = -3$ (2) $y = \frac{9}{4}$ or $y = -\frac{15}{4}$
(3) $x = 1$ or $x = -\frac{1}{7}$ (4) $x = \frac{3}{26}$
(5) $y = 5$ or $y = -5$

Exercise 6.3

(1) $x = 2, y = 3$. (2) $x = 2, y = 3$. (3) $x = 27, y = -32$.
(4) $x = 3, y = 1$. (5) $x = 1, y = -1$. (6) $x = 8, y = 2$.
(7) $x = 7, y = 5$. (8) $x = 5, y = 6$. (9) $x = 10, y = 2$.
(10) $x = 3, y = 2$. (11) $x = 15, y = 10$ (12) $x = 9, y = 20$.
(13) $x = 8, y = 5$ (14) $x = 4, y = 6$. (15) $x = 2, y = 3$.
(16) $x = 13, y = -26$. (17) $x = 3, y = 4$. (18) $x = 8, y = 2$.
(19) $x = 3, y = -3$. (20) $x = 7, y = 11$ (21) $x = 9, y = 20$.
(22) $x = 11, y = 9$. (23) $x = 11, y = 2$ (24) $x = -\frac{1}{2}, y = 3$
(25) $x = 4, y = 3$.

Exercise 6.4

(1) $x = 2, y = 1$. (2) $x = 5, y = 2$ (3) $x = 5, y = -1$
(4) $x = 3, y = 1$ (5) $x = 1, y = -5$ (6) $x = \frac{13}{5}, y = \frac{7}{5}$
(7) $x = 4, y = -2$ (8) $x = 2, y = 4$ (9) $x = 2, y = 3$
(10) $x = 2, y = -2$

Exercise 6.5

(1) $x = 6, y = 5$ (2) $x = 3, y = 2$ (3) $x = 15, y = 5$

(4) $x=-4,\ y=-6$ (5) $x=15,\ y=10$ (6) $x=12,\ y=8$
(7) $x=8,\ y=5$ (8) $x=3,\ y=4$ (9) $x=2,\ y=3$
(10) $x=15,\ y=8$

Exercise 6.6

(1) $x = 2,\ y =-1$ (2) $x = 5,\ y = -3$ (3) $x = 3,\ y = 1$
(4) $x = 1,\ y = 2$ (5) $x = 1,\ y = 2$ (6) $x = 2,\ y = 3$
(7) $x = 3,\ y =-3$ (8) $x = 1,\ y = 2$ (9) $x =-2,\ y = 3$
(10) $x = -2,\ y = 2$

Exercise 6.7

(1) $x = -1,\ y = 1$ (2) $x = 2,\ y = 3$ (3) $x = -3,\ y = 2$
(4) $x = 4,\ y = -3$ (5) $x = 2,\ y = 3$ (6) $x = 1,\ y = -3$
(7) $x = -2,\ y = 4$ (8) $x = \frac{5}{2};\ y = 2$

Miscellaneous Exercise 6 – (A)

(1) $x = 2,\ y =-5$ (2) $x = 4,\ y = 9$ (3) $x = -\frac{8}{5}$
(4) $x = 6,\ y = -4$ (5) $x = 4,\ y = 3$ (6) $x = 5$
(7) $x = 7;\ y = 11$ (8) $x = 2,\ y = 3$ (9) $x = 8,\ y = 2$
(10) $x = -2$ (11) $x = 4,\ y = 6$ (12) $x = 4,\ y = 10$
(13) $p = -15$ (14) $x = -\frac{1}{2},\ y = 3$ (15) $x = 3,\ y = 4$
(16) $x = 15,\ y = 8$ (17) $p = -\frac{7}{9}$ (18) $x = 5,\ y = 3$
(19) $x = 2,\ y = -1$ (20) $x = -13$ (21) $x = 1,\ y = 3$
(22) $x = 1,\ y = 4$ (23) $x = 12,\ y = 8$ (24) $x = \frac{17}{5}$
(25) $x = -2,\ y = 2$ (26) $x = 1,\ y = 2$ (27) $x = 2,\ y = 3$
(28) $p = \frac{8}{7}$

Exercise 6.8

(1) 84 (2) Age of the son, 20 yrs; mother, 40 yrs.
(3) 10 km./hour (4) 37 (5) 14 (6) 48
(7) 32 (8) 54 (9) Rs. 375 (10) 10 yrs.
(11) Length 30 cm., breadth 15 cm.
(12) Length 25 decimetre, breadth 10 decimetre.
(13) $m\angle A = 75^\circ,\ m\angle B = 70^\circ,\ m\angle C = 35^\circ$
(14) 8, 14, 18 cm. (15) Length 40 cm. breadth 20 cm.
(16) 20 cm.

Exercise 6.9

(1) 35 , 10 (2) 14 , 8 (3) 57 , 38 (4) 75 , 50 (5) 120 , 80 (6) 7 mtrs, 5 mtrs. (7) Father 38 yrs, Son 5 yrs. (8) Person 36 yrs. son 9 yrs. (9) Mother 45 yrs., Umesh 15 yrs.

Exercise 6.10

(1) Orange Ps. 75, apple Rs. 1. (2) Alg. Rs. 6, Geom. Rs. 5 (3) $\frac{5}{8}$ (4) 448 km. (5) 41 tickets of Rs. 10; 64 tickets of Rs. 8 (6) 100° 80°, 100°. 80°.

Exercise 6.11

(1) $m\angle A = 50^\circ$, $m\angle C = 40^\circ$ (2) Length 63 cm., breadth 27 cm. (3) 48 cm., 36 cm. (4) 84°, 96°.

Miscellaneous Exercise – 6 (B)

(1) $\frac{6}{8}$ (2) 54 (3) Rs. 200, boys 40 (4) Rs. 1500 (5) 800 (6) Tea Rs. 19.25, sugar Rs. 1·40 (7) 360 km. (8) Length 15 cm. breadth 12 cm. (9) Length 16 cm., breadth 10 cm. (10) Rs. 125 (11) 19 yrs., 24 yrs. (12) 5 (13) 32, 40 (14) 80 Paise (15) 24 kg. (16) 5 cm., 5 cm., 8 cm.

Chapter 7

Exercise 7.1

(1) $\{-2, -\frac{1}{3}\}$ (2) $\{\frac{5}{2}, 2\}$ (3) $\{-6, -\frac{3}{2}\}$ (4) $\{2, \frac{4}{3}\}$
(5) $\{2, \frac{3}{4}\}$ (6) $\{-2, -\frac{3}{4}\}$ (7) $\{-\frac{3}{2}, \frac{1}{4}\}$ (8) $\{1, \frac{6}{5}\}$
(9) $\{-\frac{3}{2}, -\frac{4}{3}\}$ (10) $\{\frac{5}{2}, \frac{2}{3}\}$ (11) $\{\frac{3}{2}, -\frac{2}{3}\}$ (12) $\{\frac{7}{2}, -1\}$
(13) $\{-\frac{5}{2}, \frac{2}{3}\}$ (14) $\{\frac{3}{2}, -1\}$ (15) $\{-3, -\frac{5}{2}\}$ (16) $\{3, -\frac{2}{7}\}$
(17) $\{-\frac{2}{3}, -\frac{1}{4}\}$ (18) $\{\frac{3}{2}, -\frac{4}{3}\}$ (19) $\{-3, -\frac{2}{5}\}$ (20) $\{3, -\frac{2}{3}\}$

Exercise 7.2

(1) $\{0, 6\}$ (2) $\{0, \frac{5}{3}\}$ (3) $\{0, -\frac{7}{4}\}$ (4) $\{0, \frac{5}{9}\}$
(5) $\{0, \frac{5}{2}\}$ (6) $\{0, \frac{4}{3}\}$ (7) $\{0, 1\}$ (8) $\{0, -4\}$
(9) $\{0, 3\}$ (10) $\{0, 1\}$ (11) $\{0, -\frac{3}{4}\}$ (12) $\{0, -\frac{5}{6}\}$
(13) $\{0, \frac{3}{2}\}$ (14) $\{0, -5\}$ (15) $\{0, 5\}$

Exercise 7.3

(1) $\{2, -2\}$ (2) $\{5, -5\}$ (3) $\{\frac{1}{3}, -\frac{1}{3}\}$ (4) $\{\sqrt{3}, -\sqrt{3}\}$

(5) $\{5, -7\}$ (6) $\{(-3+\sqrt{7}), (-3-\sqrt{7})\}$

(7) $\{3, -13\}$ (8) $\left\{\frac{1+3\sqrt{2}}{3}, \frac{1-3\sqrt{2}}{3}\right\}$

(9) $\{\frac{3}{4}, -\frac{5}{4}\}$ (10) $\{(2+\sqrt{3}), (2-2\sqrt{3})\}$ (11) $\{\frac{4}{3}, -\frac{4}{3}\}$

(12) $\{2a, -2a\}$ (13) $\{6, -2\}$ (14) $\{2, -8\}$ (15) $\{3, -6\}$

(16) $\{5, -7\}$ (17) $\{1, -11\}$ (18) $\{10, -8\}$ (19) $\{9, 5\}$

(20) $\{\frac{11}{2}, -\frac{9}{2}\}$

Exercise 7.4

(1) $\{(-3+\sqrt{11}, (-3-\sqrt{11})\}$ (2) $\{(2+\sqrt{3}), (2-\sqrt{3})\}$

(3) $\left\{\left(\frac{-7+\sqrt{13}}{6}, \frac{-7-\sqrt{13}}{6}\right)\right\}$ (4) $\left\{\frac{1+\sqrt{11}}{5}, \frac{1-\sqrt{11}}{5}\right\}$

(5) $\left\{\frac{7+\sqrt{73}}{12}, \frac{7-\sqrt{73}}{12}\right\}$ (6) $\left\{\frac{-3+\sqrt{13}}{2}, \frac{-3-\sqrt{13}}{2}\right\}$

Exercise 7.5

(1) 3 (2) 7 (3) $\frac{3}{2}$ or $\frac{2}{3}$ (4) 9 or $\frac{1}{9}$ (5) 6, 8 (6) 11, 13

(7) 12, 15 (8) 40 yrs. 5yrs. (9) 93 (10) 11, 13, 15.

Exercise 7.6

(1) Length 50 mtr. breadth 22.5 mtr. (2) Length 48 mtr.

(3) Length 12 cm. , breadth 12 cm. (4) Length 8 mtr., breadth 6 mtr. (5) Length 18 mtr. , breath 12 mtr. (6) 11 mtr.

(7) Length 12 mtr. , breadth 10 mtr . (8) 6 cm. and 8 cm.

(9) 24 cm. and 18 cm. (10) 7 cm. and 24 cm.

Exercise 7.7

(1) Length 15 cm. , breadth 5 cm. (2) Altitude 6 cm. , base 15 cm

(3) 5 cm. , 7 cm. (4) 7 cm. and 12 cm.

Miscellaneous Exercise – 7

(1) 9 and 6 (2) 17 (3) 11 (4) Length 27 m. breadth 16 m.

(5) 15 (6) Two, 45 and 54 (7) 20 (8) 480 sq. m.

(9) 30 (10) 25 cm. (11) 15 and 17 (12) 16 (13) 20

(14) Two, $\frac{3}{4}$ and $\frac{4}{3}$ (15) 7

Chapter 8

Exercise 8.1

1. (1) $5x^2 + 6x + 12$ (2) $5a^2 - 3a + 1$ (3) $9a^2b - 4ab^3 + 3b^4$
(4) $2a^4$ (5) 0 (6) $5x^3 - 3x^2 + 8$
(7) $x^4 + 2x^3 - x + 1$ (8) $x^2 - \frac{7}{6}x + 2$ (9) $10x^2 + 6x + 9$
(10) 0

2. (1) $x^3 - 6x^2 + 12x + 4$. (2) $7a^2 + 3a + 1$ (3) $-4b^2 - 6b$
(4) $-8a^3b + 9a^2b^2 + ab^3 - 3b^4$ (5) $\frac{1}{3}x^2 - x - \frac{20}{3}$
(6) $2a^2 - 5a + 7$ (7) $2n^2 - mn$ (8) $y^2 - 5xy - x^2$
(9) $x^3 - 5x + 6$ (10) 0

3. (1) $2x^2 - 6x + 1$ (2) $-4a + 6$ (3) $2m^2 - 2m + 5$
(4) 2 (5) $4a + 3b + 5c$ (6) $4x^2 - 17x - 13$
(7) $-y^2 - 5y + 10$ (8) $4y - 2z$

4. $2x^2 + 9x - 8$ 5. $-a^2 - 2a$ 6. $-5m + 12$

7. (1) $7x^2 - 7x + 13$ (2) $x^2 - 2x + 10$ (3) $x^2 - 7x - 8$
(4) $-4x^2 + 9x - 9$ (5) $7x^2 - 7x + 13$ (6) $3x^2 - 7x - 1$
(7) $6x^2 - 9x + 16$ (8) 0 (9) $2x^2 + 7$
(10) $x^2 + 2x - 3$ (11) $-5x^2 + 7x - 6$ (12) $5x^2 - 7x + 6$

Exercise 8.2

1. (1) 2 (2) 1 (3) 0 (4) 3 (5) no degree
(6) 3 (7) n (8) $m + n$

2. (1) 5 (2) -4 (3) 1 (4) 5 (5) -1

3. (1) -5, 3 (2) x^{-3}, because the index -3 is not a whole number
(3) $x^{\frac{1}{2}}$, because the index $\frac{1}{2}$ is not *a* whole number (4) 0
(5) 7 (6) 4 (7) $-5m$

Exercise 8.3

7. (1) 2 ; (3, − 10, 7) (2) 4 ; (1, 7, 0, 7, −6)
(3) 4 ; (1, 0, 0, −5, 0) (4) 4, ; (1, 0, 0, −5, 7)
(5) 5 ; (−7, 0, 0, 0, 0, 0)(6) 0 ; (−5)
(7) 3 ; (1, 0, $-p$, q) (8) 1 ; (1, 0)
(9) 2 ; (a , 0, b) (10) 1 ; (−1, 0)

(11) $3; (a, 0, -d, 0)$ (12) $1; (a, -c)$

2. (1) $2;\ x^2 + 10x - 5$ (2) $4;\ 5y^4 - 7$
   (3) $1;\ 4x - 3$ (4) $4;\ 2x^4$
   (5) $3;\ ax^3 + bx^2 + cx + d$ (6) $3;\ ay^3 + b$
3. $a = 4$, $b = -5$, $c = 7$ and $d = -6$
4. $a = 7$, $b = -5$ and $c = -4$; $a+b+c = -2$

Exercise 8.4

1. (1) $x^3 + 8x^2 + 17x + 6;\ 3$ (2) $5x^3 - 12x^2 + 7x - 6;\ 3$
   (3) $a^5 - a^4 - 13a^3 + 28a^2;\ 5$ (4) $m^6 + 5m^4 - m^2 - 5;\ 6$
   (5) $y^4 - 1;\ 4$ (6) $x^3 + 1;\ 3$
   (7) $5x^4 - 15x^3 - 7x^2 + 27x - 18;\ 4$ (8) $x^5 + 4x^3 + 5x^2 - 12x - 10,\ 5$
   (9) $4y^4 - 10y^3 + 5y^2 + 5y - 14;\ 4$ (10) $ax^3 + (b-ap)x^2 + (c-bp)x - cp;\ 3$
2. $x^2 + 5x - 7$; $y^3 - 3y + 4$; $a^2 - 7a + 8$; $x^3 - 5x^2 + 7x - 9$,
   0 ; 0 ; 0 ; 0.
   (1) A (2) 0

Exercise 8.5

dividend = divisor × quotient + remainder,

(1) $8x^2 + 4x = (x+2) \times (8x - 12) + 24$
(2) $y^3 - 9 = (y + 3) \times (y^2 - 3y + 9) - 36$
(3) $m^3 + 64 = (m + 4) \times (m^2 - 4m + 16) + 0$
(4) $2a^2 + 3a + 1 = (a + 2) \times (2a - 1) + 3$
(5) $x^3 - 4x^2 + 5x - 2 = (x - 1) \times (x^2 - 3x + 2) + 0$
(6) $x^2 - x - 2 = (x - 1) \times x - 2$
(7) $4x^2 + 16x = (4x + 12) \times (x + 1) - 12$
(8) $2x^4 + x^3 - 19x^2 + 5x + 15 = (x^2 - 7) \times (2x^2 + x - 5) + (12x - 20)$
(9) $125y^3 + 1 = (5y + 1) \times (25y^2 - 5y + 1) + 0$
(10) $x^4 + x^2 + 1 = (x^2 + x + 1) \times (x^2 - x + 1) + 0$
(11) $3x^2 + x - 10 = (x + 2) \times (3x - 5) + 0$
(12) $y^3 + 10y^2 - 48y + 1 = (y^2 - 5y + 7) \times (y + 15) + (20y - 104)$
(13) $x^2 - a^2 = (x + a) \times (x - a) + 0$
(14) $x^3 + a^3 = (x + a) \times (x^2 - ax + a^2) + 0$

(15) $ax^2 + bx + c = (x - p) \times (ax + b + ap) + (ap^2 + bp + c)$

(16) $ax^2 + bx + c = (x + p) \times (ax + b - ap) + (ap^2 - bp + c)$

Exercise 8.6

(1) $x^3 - 5x^2 + 7x + 2 = (x - 1)(x^2 - 4x + 3) + 5$

(2) $x^4 - 5x^2 + 3 = (x - 1)(x^3 + x^2 - 4x - 4) - 1$

(3) $y^3 + 7y^2 + 5y + 3 = (y + 1)(y^2 + 6y - 1) + 4$

(4) $y^4 - 7y^2 + 2 = (y - 1)(y^3 + y^2 - 6y - 6) - 4$

(5) $x^4 - x^2 + 11x - 5 = (x - 1)(x^3 + x^2 - 6x + 5) + 0$

(6) $y^3 + 7y^2 + 9y + 3 = (y + 1)(y^2 + 6y + 3) + 0$

(7) $x^4 - 16 = (x - 2)(x^3 + 2x^2 + 4x + 8) + 0$

(8) $y^4 - 16 = (y + 2)(y^3 - 2y^2 + 4y - 8) + 0$

(9) $x^3 + 125 = (x + 5)(x^2 - 5x + 25) + 0$

(10) $x^3 - 7x + 3 = (x - 3)(x^2 + 3x + 2) + 9$

(11) $4x^4 - 5x^3 + 15 = (x - 3)(4x^3 + 12x^2 + 31x + 93) + 294$

(12) $5y^4 - 2y^3 + 7y - 5 = (y - 2)(5y^3 + 8y^2 + 16y + 39) + 73$

(13) $x^3 + 0x^2 + 0x - a^3 = (x - a)(x^2 + ax + a^2) + 0$

(14) $x^3 + a^3 = (x + a)(x^2 - ax + a^2) + 0$

(15) $ax^2 + bx + c = (x - p)(ax + b + ap) + (ap^2 + bp + c)$

(16) $ax^2 + bx + c = (x + p)(ax + b - ap) + (ap^2 - bp + c)$

Exercise 8.7

is divisible : (1), (2), (3), (4), (6), (7), (8), (10), (11), (12).

is not divisible : (5), (9).

Exercise 8.8

1. (1) -4 (2) 4 (3) -2 (4) $3a^2 + 5a - 4$ (5) $3p^2 - 5p - 4$

2. (1) -6 (2) 9 (3) $-\frac{15}{8}$ (4) $-8a^3 - 4a - 3$

(5) $(a + b)^3 + 2(a + b) - 3$

3. (1) 0 (1) $-8a^3$

4. $pa^2 + qa + r$

5. $8a^3 + 7a^2 - 10a$

6. $\left(a + \frac{1}{a}\right)^2 - 2\left(a + \frac{1}{a}\right)$

7. $c = -8$

8. $a = 0$, $b = -5$

9. $p = -\frac{10}{3}$, $q = -\frac{26}{3}$

10. $b = 0$, $c = 7$

Exercise 8.9

1. (1) 20 (2) 0 (3) -6 (4) 21 (5) 22
(6) 0 (7) 0 (8) 0 (9) 11 (10) -4

2. $a = 3$ 3. $b = 3$ 4. $a = 2$ 5. $p = 16$

6. $a = 5$; $b = -10$ 7. $P = -4$

8. remainder $= 0$, is divisible. $x^2 - 5x + 4 = (x-1)(x-4)$

Exercise 8.10

1. (1) is a factor, $x - 4$ (2) is a factor, $x^2 - 4x + 3$
(3) is a factor, $x^2 + 5x + 6$ (4) is a factor, $x^2 - 2x + 8$
(5) is a factor, $2y^2 - y + 5$ (6) is a factor, $m^2 - 2m + 1$
(7) is a factor, $p^2 - 7p - 5$ (8) is a factor, $y^2 - y - 1$
(9) is a factor, $x^3 - 4x^2 + 2x + 1$ (10) is a factor, $x^3 - 5x + 1$

2. $p = 10$ 3. $p = 5$, $x = 3$ 4. $a = 7$, $x + 4$

5. $a = 16$ 6. $a = 7$, $b = 6$ 7. $p = -7$, $q = -5$

8. $a = -1$, $b = -9$ 9. $a = 1$, $x^3 - x^2 + 6x + 1$

10. $b = -10$, $4x^3 + 3x^2 - 4x - 8$

Exercise 8.11

1. is a factor. $x^2 - 4x - 3$ 2. is a factor. $2x^2 - 5x - 10$
3. is a not factor. 4. is a factor. $3x^2 + 3x + 1$
5. is a factor. $x^2 - x + 4$ 6. is a factor. $2x - 1$
7. is a factor. $x^2 + x - 26$ 8. is a factor. $x^3 + x^2 + x + 1$
9. is a not factor. 10. is a factor. $5x^3 + 11x^2 + 11x + 11$.

Exercise 8.12

1. is a factor. $x + 4$ 2. is a factor. $2x^2 - 7x + 4$
3. is a factor. $x^2 - 6x + 9$ 4. is a factor. $2x^2 + 3x + 1$
5. is a factor. $x^2 + 4x + 2$ 6. is a factor. $x^2 - 6x - 1$
7. is a factor. $x^2 - 6x + 12$ 8. is not a factor.
9. is a factor. $x^3 - x^2 + x -$ 10. is a factor. $ax^2 + (b-a)x + (a+1)$

Exercise 8.13

(1) $(x-1)(x+1)(x+2)$ (2) $(y-1)(y-2)(y+3)$
(3) $(m+1)(m-5)(m+4)$ (4) $(x-1)(x-1)(x-3)$
(5) $(y+1)(y-2)(y-3)$ (6) $(y-1)(y+1)(y+3)$

(7) $(x+1)(x^2+5x+1)$ (8) $(x+1)(x+2)(x+3)$
(9) $(m+1)(m+5)(m+6)$ (10) $(p+1)(p-7)(p+5)$
(11) $(y+2)(y^2-y+4)$ (12) $(x+2)(x+3)(x+4)$
(13) $(x-2)(x^2+x+1)$ (14) $(x-3)(x-4)(x+7)$
(15) $(m-3)(m^2+4m+12)$ (16) $(x-2)(x-8)(x+7)$
(17) $(x+3)(x+5)(x-8)$ (18) $(y-3)(y-7)(y+10)$
(19) $(x-1)(x+1)(2x-1)$ (20) $(x-4)(x+4)(x^2+3)$
(21) $(x+1)(2x+3)(x-5)$ (22) $(x-1)(x+1)(x^2+4x+7)$
(23) $(x-1)(x-2)(x+1)(x+2)$ (24) $(y-1)(2y+3)(3y+2)$

Exercise 8.14

(1) $12x$ (2) $3x$ (3) xz
(4) $x-1$ (5) $(y-1)^2$ (6) $(x-y)(x+2y)$
(7) $3(x-1)$ (8) $2(x+y)$ (9) $x+3$
(10) $x+3$ (11) $2(x-2y)$ (12) $x-1$

Exercise 8.15

1. (1) $36a^3b$ (2) a^3b^4
(3) $(x-a)^2(x+a)^2$ (4) $(y-1)^2(y-2)(y+3)$
(5) $(a-b)^2(a-2b)(a+2b)^2$ (6) $(m+2)^2(m-3)$
(7) $(x-2)(x-3)(x+2)(x+3)$ (8) $12x^2(x-2)(x+2)$
(9) $10(x-2y)^2(x+2y)$ (10) $(x-2)(x-3)(x+3)$

2. (1) $x+3$; $(x+3)(x+2)(x-4)$
(2) $x-y$; $(x-y)(x+y)(x^2+xy+y^2)$
(3) $y-4$; $(y-4)(y+4)(y+3)$
(4) $x-1$; $2(x-1)^2(x^2+x+1)$
(5) $m+4$; $(m+4)(m+3)$
(6) $a+2$; $(a+2)^2(a-2)$
(7) x^2-3x+9 ; $2(x+3)(x^2-3x+9)$
(8) $(a+b)(a-b)$; $(a+b)(a-b)(a^2+b^2)$
(9) $y-2$; $y(y-2)(y+1)(y+2)$

(10) $x-4$; $(x-4)(x+5)(x+6)$

(11) $x-2$; $(x-1)(x-2)(x-3)(x^2+2x+4)$

(12) $x-1$; $x(x-1)(x-3)(x+1)$

(13) $x+7$; $(x+7)(x-4)(x^2-7x+49)$

(14) $y+1$; $(y+1)(y+2)(y^2+5y+1)$

(15) $x-1$; $(x-1)^2(x-2)(x+2)(x+3)$

(16) $(m-1)(m-2)$; $(m-1)^2(m-2)(m+3)$

(17) $(x+1)(x-1)$; $(x+1)(x-1)(x^2+1)(x^4+1)$

(18) $(x-4)$; $2(x-4)(x-5)(x+4)(x^2+4x+16)$

(19) $7(m-1)$; $42(m-1)^2(m+1)(m^2+m+1)$

(20) $x-2y$; $(x-2y)(x-y)(x+y)(x+2y)$

(21) $2(x+2)$; $24(x+2)(x+3)(x-2)(x-3)$

(22) a^2-a+1 ; $(a^2-a+1)(a^2+a+1)(a+1)(a-2)$

(23) $2x-3$; $(2x-3)(2x+3)(x+5)(x-5)$

(24) $2(y-5)$; $8(y-5)(y+5)(y-2)(y^2+5y+25)$

(25) $2(x+3)$; $8(x+3)(x-3)(x-2)^2$

Miscellanous Exercise – 8

1. (1) $x+18$ (2) $a+b+c$ (3) $m-23n$ (4) 0

2. (1) y^3-1 ; 3 (2) m^4+m^2+1 ; 4

(3) $2a^3-9a^2+17a-12$; 3 (4) $3x^3-11x^2-11x+35$; 3

3. (1) $x^3-4x^2+5x-3=(x-5)(x^2+x+10)+47$

(2) $y^3-3y^2+7=(y^2-2)(y-3)+(2y+1)$

(3) $4m^3-5m^2+7m=(m+3)(4m^2-17m+58)-174$

(4) $3x^3-5x^2+7x-28=(x-2)(3x^2+x+9)-10$

4. $p=14$

5. (1) $(x-3)(x-5)(x-6)$ (2) $(y-3)(y-5)(y+8)$

(3) $(m+2)(m^2-m+4)$ (4) $a(a-2)(a^2+2a+4)$

(5) $(x-2)(x+2)(x+3)$ (6) $(x-1)(x+1)(x+5)$

(7) $a(a-2)(a+3)$ (8) $(x+2)(x^2-2x+2)$

(9) $(x-1)(x+1)(x^2+1)$ (10) $(a-1)(a-3)(a+2)$

6.

No	G. C. D.	L. C. M.
(1)	$3x(x-1)$	$6x^2(x-1)^2(x+1)(x+2)$
(2)	$(x-2y)$	$2(x-2y)^2(x^2+2xy+4y^2)$
(3)	$(x+1)(x+2)$	$(x+1)(x+2)(x+3)(x-3)$
(4)	$2(a-b)(a+b)$	$12(a-b)(a+b)(a^2+b^2)$
(5)	$x+7$	$3(x+7)(x-7)(x^2-7x+49)$

7. (1) is a factor (2) is not a fctor
(3) is not a factor. (4) is a factor.

8. (1) 1 (2) -2 (3) -3 (4) -3

9. (1) $x^3-4x^2+5=(x-5)(x^2+x+5)+30$
(2) $y^3-5y^2+5y-1=(y-1)(y^2-4y+1)+0$
(3) $x^3-7x^2+5x-4=(x+2)(x^2-9x+23)-50$
(4) $2m^4-3m^2+5m-15=(m+3)(2m^3-6m^2+15m-40)$
$+105$

10. $a=5$, $b=3$

Chapter 9

Exercise 9.1

(2) meaningful ; 0 (3) meaningful; -4 (4) meaningless
(5) meaningful ; 0 (6) meaningless.

Exercise 9.2

(1) $\frac{5}{7}ab$ (2) $\frac{x+2y}{x-4y}$ (3) $\frac{a+b}{a-b}$ (4) $x-1$

(5) $\frac{x}{y}$ (6) $\frac{x+3}{x+1}$ (7) $\frac{a^2-ab+b^2}{a+3b}$ (8) $\frac{x}{2y}$

(9) $\frac{x^2+4x-1}{x-4}$ (10) $\frac{y^2+2y-6}{y-5}$

Exercise 9.3

(1) $\frac{m+3}{m-3}$ (2) x (3) $2(x-y)$ (4) $\frac{m-n}{m+n}$

(5) 1 (6) 1 (7) $p^2 - 1$ (8) $p - m$ (9) 1

(10) b^2 (11) $\frac{(a-4)(a-8)(a+6)}{(a-3)(a-6)(a+8)}$ (12) $\frac{x+5}{x+4}$

(13) 1 (14) 1 (15) $\frac{q(6-q)}{(q+2)(q-2)}$ (16) $\frac{x^2+xy+y^2}{x^2-y^2}$

(17) $\frac{2(m^2-11m+26)}{(m-7)(m-3)}$ (18) $\frac{2x}{x+2}$ (19) $\frac{1}{(a+4)(a+5)}$

(20) $\frac{b}{(a+4b)(a+5b)}$ (21) $\frac{1}{1+a}$ (22) $\frac{p-4}{p^2-4}$ (23) qr

(24) $m+n$ (25) 1 (26) 1 (27) $-\frac{a}{14}$ (28) $\frac{2x}{3+x}$

(29) $\frac{a^2-1}{a^2+1}$ (30) $\frac{a-b}{a+b}$ (31) 2 (32) $\frac{x^2}{x+3}$

(33) $\frac{x}{3x-5}$ (34) $\frac{-1}{x(x+h)}$ (35) $-\frac{(2x+h)}{x^2(x+h)^2}$

2. (1) $-\frac{14a+3}{2a+4}$ (2) $\frac{6-x}{3x-2}$ (3) $-\frac{3}{2}m$

3. (1) 1 (2) $-\frac{1}{51}$ (3) 1

Exercise 9.4

1. $x = 6$ 2. $m = 2$ 3. $x = \frac{11}{2}$ 4. $x = 5$

5. $y = -\frac{9}{2}$ 6. $m = -\frac{9}{8}$ 7. $y = 2$ 8. $y = -10$

9. $x = 8$ 10. $x = \frac{11}{2}$ 11. $x = \frac{9}{2}$ 12. $x = -10$

13. $y = 9$ 14. $x = \frac{9}{2}$ 15. $x = -\frac{3}{2}$ 16. $x = -5$

17. $y = 4$ 18. $x = -\frac{3}{2}$ 19. $y = -3$ 20. $x = 5$

Miscellaneous Exercise – 9

1. (1) $5x^2y$ (2) $\frac{m-2n}{m+3n}$ (3) $\frac{a^2+ab+b^2}{(a+b)(a^2+b^2)}$

(4) $\frac{x-2}{x-1}$ (5) $\frac{a-3}{a+4}$ (6) $\frac{x^2+4x+3}{x(x^2-x+1)}$

2. (1) $\frac{a-3}{a+3}$ (2) $\frac{(x+3)(x+7)}{x-7}$ (3) $\frac{1}{x^2+3x-4}$

(4) 1 (5) 3 (6) $\frac{m}{6(m-1)(m-3)}$

(7) $\frac{1}{2m-1}$ (8) $\frac{x-7}{x-10}$ (9) $\frac{p-1}{p}$

(10) $3x^2 + 3xh + h^2$

3. (1) $x = -\frac{7}{2}$ (2) $y = 4$ (3) $m = \frac{11}{6}$ (4) $x = \frac{-11}{2}$

(5) $y = \frac{9}{2}$ (6) $x = 1$ (7) $y = \frac{11}{2}$ (8) $x = -2$

(9) $x = -4$ (10) $p = \frac{4}{3}$

Chapter. 10

Exercise 10.1

1 (i) $\frac{3}{5}$, $y = \frac{3}{5}x$. (ii) $\frac{22}{7}$, $y = \frac{22}{7}x$. (iii) $\frac{4}{3}$, $y = \frac{4}{3}x$.

(iv) $\frac{13}{8}$, $y = \frac{13}{8}x$. (v) $\frac{5}{12}$, $y = \frac{5}{13}x$. (vi) $\frac{13}{5}$, $y = \frac{13}{5}x$

2. (i) 50, (ii) 30.

3. (i) 125, (ii) 8.

4. 20 cub. cm.

5 Rs. 220

Exercise 10.2

1. (i) 150, $xy = 150$. (ii) 15, $xy = 15$. (iii) 8, $xy = 8$

(iv) 2.88, $xy = 2.88$. (v) 1000, $xy = 1000$. (vi) 180, $xy = 180$.

2. (i) 8, (ii) 0.4.

3. (i) $\frac{16}{3}$, (ii) $\frac{1}{4}$.

4. 50 km. p. h.

5. 4 hours

6. (1) 40 mm. (2) 30 cub. cm. (3) inverse, 960, (4) yes, $8\sqrt{15}$

Miscellanous Exercise - 10

1. (i) 3, (ii) 128.

2. $\frac{4}{3}$. 3. $\frac{1}{64}$. 4. 6.

5. 21 cm. 6. 2.5 km. 7. $6\frac{3}{4}$ dynes.

8. Rs. 784 9. Rs. 750.

Chapter 11

Miscellaneous Exercise - 11

1. Increase of Rs. 4   2. Rs. 320, 6 $\frac{14}{31}$%.   3. 125, 6·4%.
4. Loss, Rs. 18   5. Hakimbhai's, by Rs. 5   6. Profit of Rs.24.
7. In 9% at 145, 8. Rs. 1470·50   9. Nothing. 10. Profit of Rs. 252.

Chapter 12

Miscellaneous Exercise - 12

1. (i) 8·6   (ii) 510
2. (i) 8·6   (ii) 510
3. (i) 16   (ii) 78
4. (i) median 95 (ii) mode 98
5. (i) 83·8   (ii) 83·8
6. Mahipatrao's
7. Rs. 26·75
8. (i) Rs. 22.90   (ii) Rs. 20.

● ● ● ● ●

MAHARASHTRA STATE BOARD OF SECONDARY
AND HIGHER SECONDARY EDUCATION PUNE

इंग्रजी बीजगणित दहावी

Rs. 16.70